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Statistical analysis of data processing in some seismic refraction methods: A synthetic data example

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The delay time method has gained attention in shallow seismic refraction survey because it has the capability to map the lateral thickness of overburden and relief of bedrock. This study addresses the comparison between the performances of the plus-minus and conventional reciprocal methods using a synthetic data. The interpretations obtained from both methods are reasonably comparable to the actual geophysical models. This suggests that either of the methods can be used to construct a geologic section. However, the result of randomized complete block design (RCBD) experiment shows a significant difference in the type of method used and this necessitate the need for further test. The pairwise comparison suggests that the plus-minus method produces a model that better mimics the actual data than the conventional reciprocal method.

Keywords: Seismic refraction, plus-minus method, conventional reciprocal method, RCBD

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1. Introduction

Shallow refraction method has been widely used for decades to evaluate near surface problems (Whiteley and Eccleston, 2006). The method has been utilized to provide information such as depth to water table, thickness of overburden, depth to consolidated materials and other parameters for geotechnical engineering, hydrogeological and environmental applications.

The delay time methods such as plus-minus (Hagedoorn, 1959) and conventional reciprocal methods (Hawkins, 1961) offer the flexibility to map slightly dipping interfaces of less than 10^o. This is an advantage over the slope intercept method which assume that the interface between layers are nearly planar. The delay time methods allow measurements in both directions from which parameters such as the dip, critical angle, seismic velocities and other parameters can be determined effectively.

In this paper, we present results of our statistical analysis of the efficiency and performance of plus-minus and conventional reciprocal methods in mapping slightly dipping reflector.

2. Method

Plus-minus and conventional reciprocal methods assume that first-arrivals only originate by critical refraction from laterally continuous refractors with relatively simple velocity distributions. Since the concepts are well known, the mathematical framework of the methods are briefly illustrated and explained. The generalized reciprocal method (GRM) (Palmer, 1981) is not considered here due to query by several authors in practically estimating the so called optimum migration distance (Sjögren, 2000; Leung, 1995; Whiteley, 1990).

2.1. Plus-minus method

The plus-minus method is derived considering the geometry shown in Fig. 1. The schematic diagram illustrates two shots at A and C. Let the delay time at the two shot points and geophone G be denoted as t_A, t_C and t_G respectively.

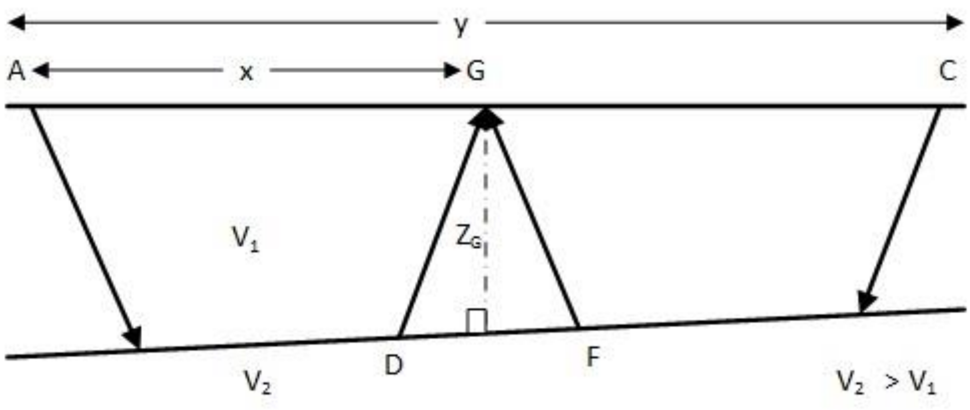


Figure 1: Geometry of refracted raypath traveling from shot points A and C to geophone placed at G for a 2 layer case:

$$t_{AC} = \frac{y}{V_2} + t_A + t_C \quad \dots \quad 1$$

$$t_{AG} = \frac{x}{V_2} + t_A + t_G \quad \dots \quad 2$$

$$t_{CG} = \frac{y - x}{V_2} + t_C + t_G \quad \dots \quad 3$$

where refraction arrival times from shot points A and C to geophone at G are denoted as t_{AG} and t_{CG} respectively, and t_{AC} is the reciprocal time for wave traveling from shot point A to a receiver at C. Adding equations 1 and 2 gives the plus term (t^+) as:

$$t^+ = \frac{1}{2}(t_{AG} + t_{CG} - t_{AC}) \dots \dots \dots \dots \dots \dots \dots \dots 4$$

The depth to the refractor under G can be computed by:

$$Z_G = \frac{t^{+}V_1V_2}{\sqrt{(V_2^2 - V_1^2)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..5$$

The minus term involves subtracting equation 3 from 2 as:

$$t_{AG} - t_{CG} = \frac{2x}{V_2} - \frac{y}{V_2} + t_A - t_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots..6$$

$$t^{-} = \frac{2x}{V_2} + K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

The velocity V_2 can then be estimated by plotting t_{AG} - t_{CG} against x.

2.2 Reciprocal method

The conventional reciprocal method computes subsurface parameters as:

$$t_V = \frac{1}{2}(t_{AG} - t_{CG} + t_{AC}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.8$$

$$t_m = \frac{1}{2}(t_{AG} + t_{CG} - t_{AC}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.9$$

where t_V is velocity analysis function, t_m is time model function. Time model function can be generally written in terms of depth, Z_G, as:

$$t_m = \sum_{j=1}^{n-1} Z_{Gj} \frac{\sqrt{V_n^2 - V_j^2}}{V_nV_j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 10$$

The time model function is converted to a depth model by multiplying by depth conversion factor (DCF) as:

$$Z_G = t_G * DCF \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..11$$

where

$$DCF = \frac{V_nV_j}{\sqrt{V_n^2 - V_j^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 12$$

where V_n and V_j are velocity of nth layer and that above it respectively, and Z_G is the perpendicular depth to the refractor under geophone G..

3. Synthetic data example

In this section, we demonstrate the accuracy of subsurface parameters (depth and velocity) of plus-minus and conventional reciprocal methods for slightly dipping reflector synthetic model (Fig. 2a) obtained by ray tracing. The model consists of two layers of 1,000 m/s and 5,000 m/s respectively. Reversed survey consisting two sources and twelve (12) geophones was utilized to simulate the refraction time arrivals. A distance of 5 m was added to both ends of the model to reduce edge effect in simulating arrival times. The time-distance curve of refraction events is shown in Fig. 2b.

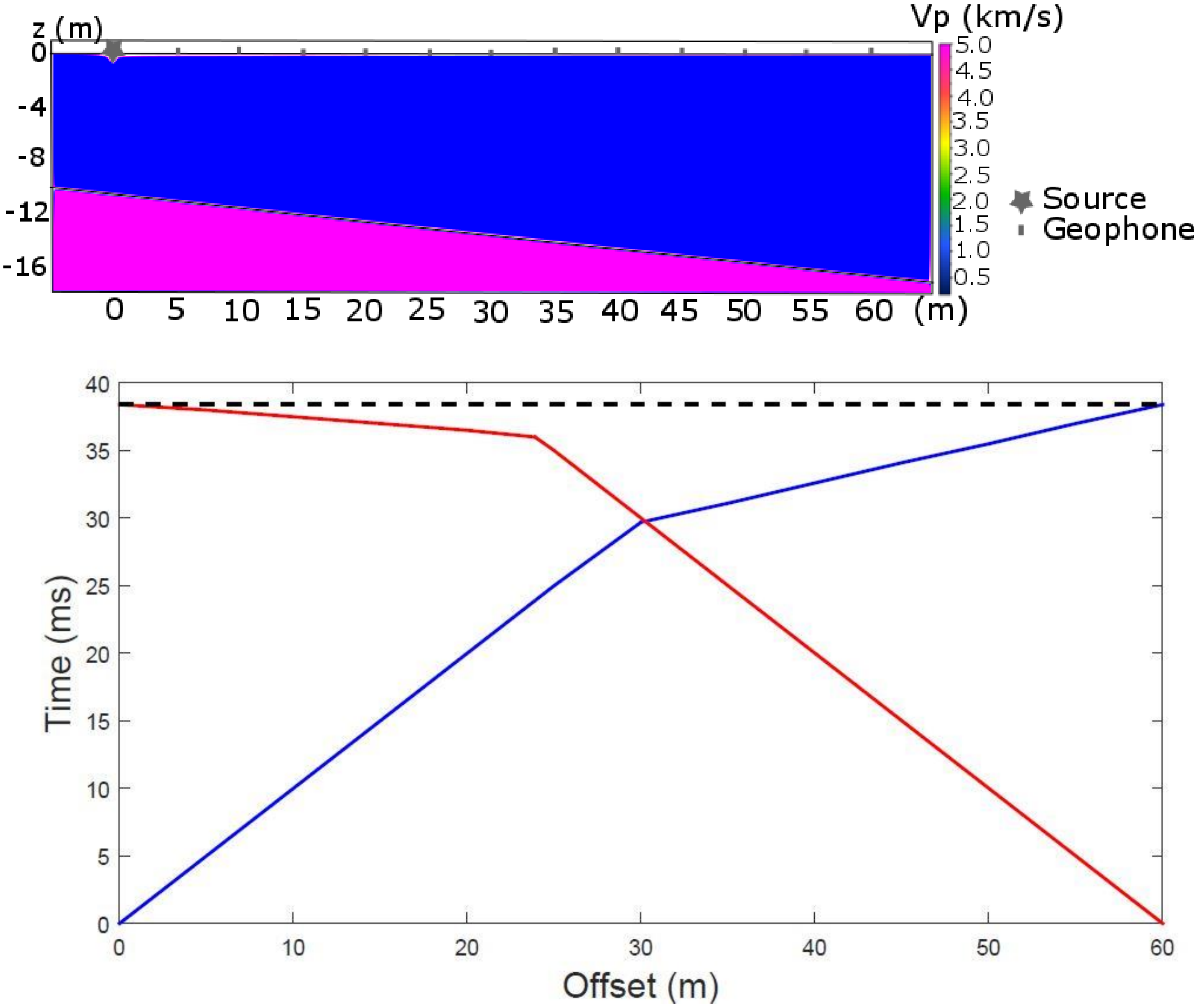


Figure 2. (*a)* Synthetic model consisting of two homogeneous velocity layers (*b*) Time-distance plot of arrival times

In both methods, the apparent velocity of the first layer is computed with intercept slope method, while the true velocity is taken as the average of apparent velocities for the two direct arrival segments from forward and reversed shooting. Figure 3 shows the minus time plot which gives the velocity of the second layer as 5,151 m/s. The depths under each geophone computed are shown in Table 1.

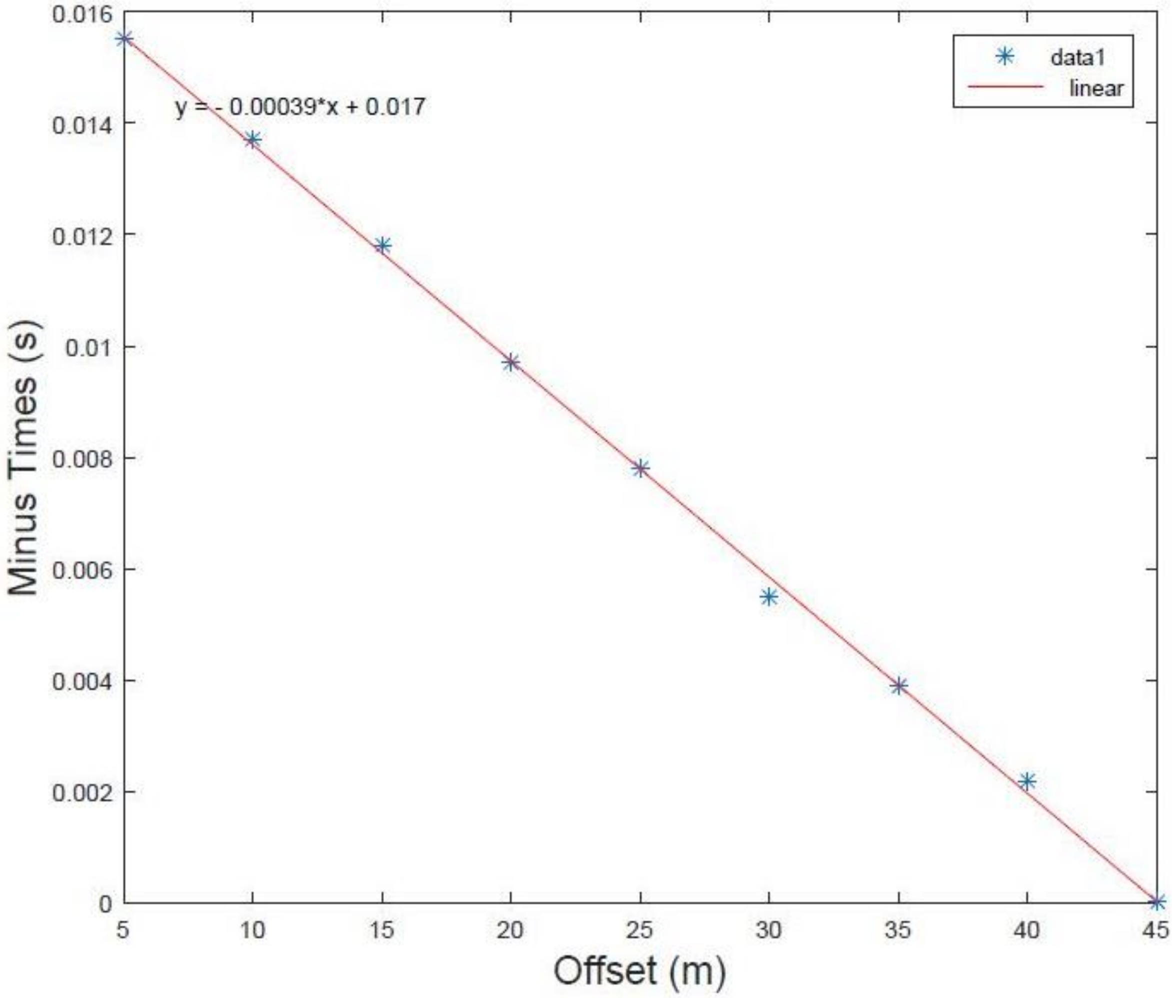


Figure 3. Time minus function of the plus-minus method.

Table 1. Computed depth in comparison to true measurement.

Geophone Position	Actual Depth	Plus-Minus Depth	Reciprocal Depth
0	10.6000	11.2200	11.2200
5	11.0000	11.3199	11.3104
10	11.3000	11.7297	11.7199
15	12.0000	12.1907	12.1805
20	12.3000	12.7541	12.7434
25	13.0000	13.2151	13.2040
30	13.3000	13.8810	13.8694
35	14.0000	14.1883	14.1764
40	14.3000	14.8542	14.8417
45	15.0000	15.2639	15.2512
60	16.3000	16.5000	16.3000

The velocity analysis function component (Fig. 4a) of the conventional reciprocal method estimates the second layer velocity as 5,263 m/s. For this method, the depth under each geophone was computed using the time model function in Figure 4b and converted to a depth section using appropriate depth conversion factor (DCF).

The depth sections (Fig. 5) generated by the two methods compare favorably with the actual model, although the depths are greater than their true values but they are within acceptable error levels, and they both exhibit similar statistics with the original data (Fig. 6).

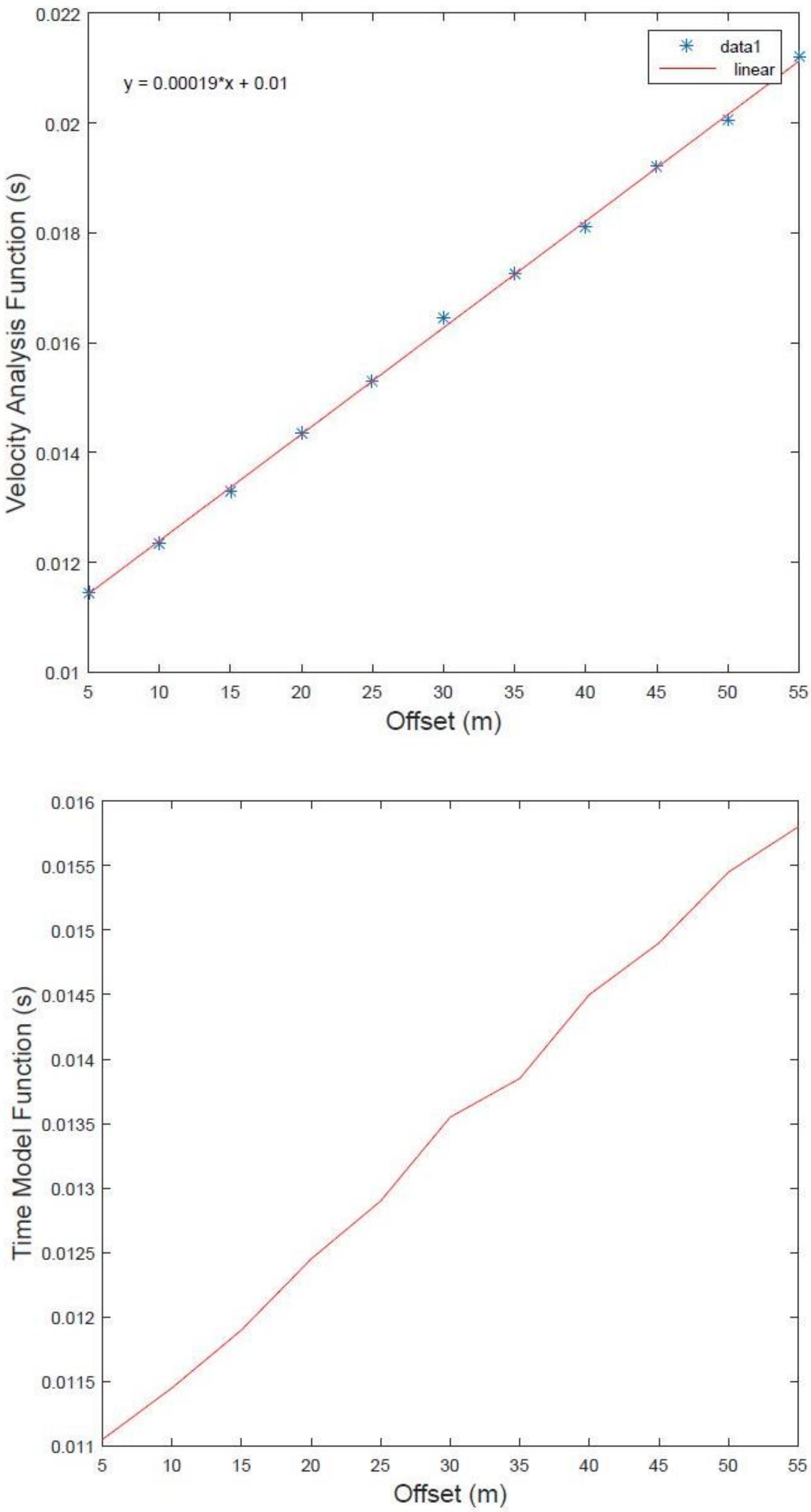


Figure 4. (*a*) Velocity analysis function plot (*b*) Time model function plot.

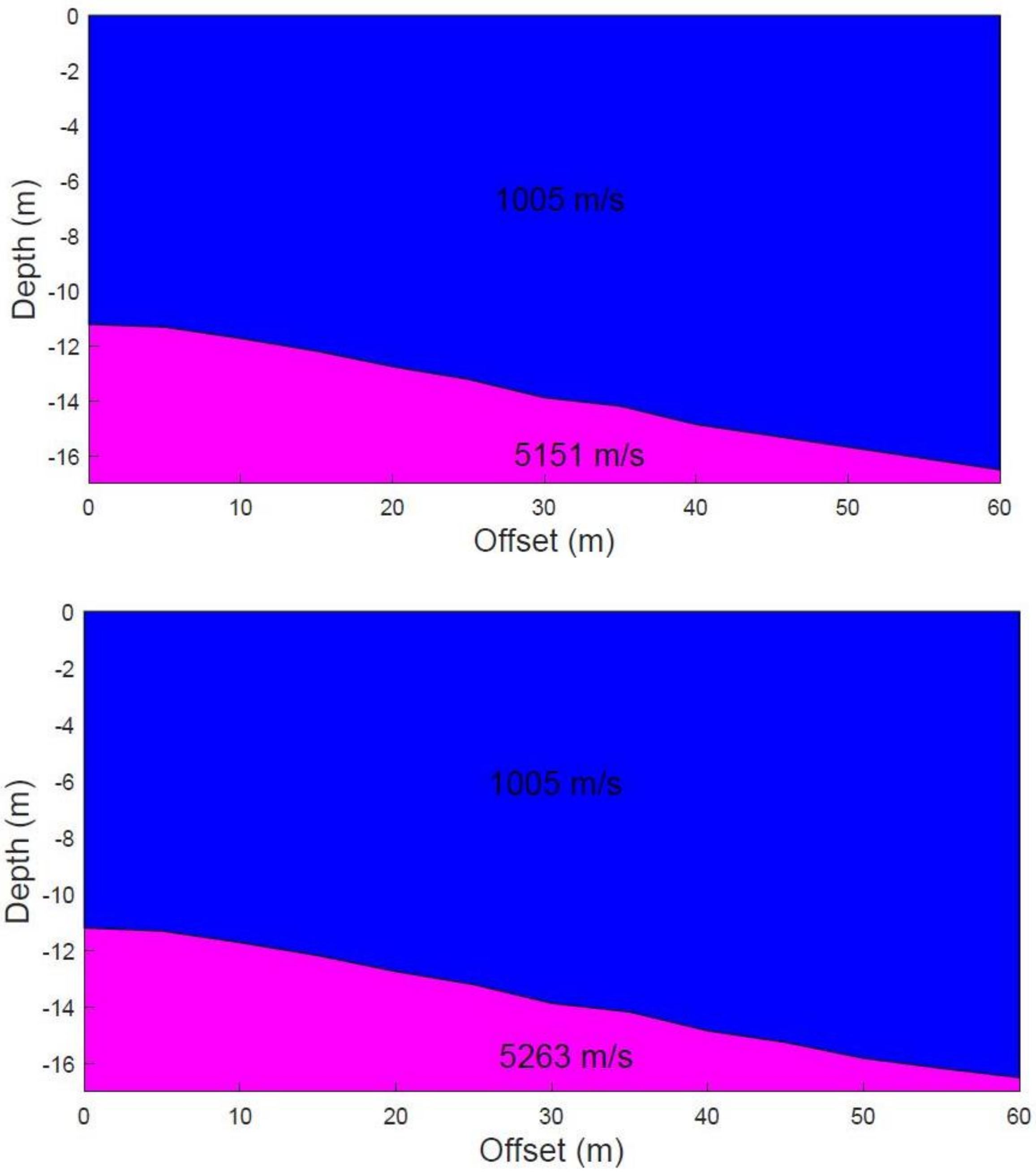
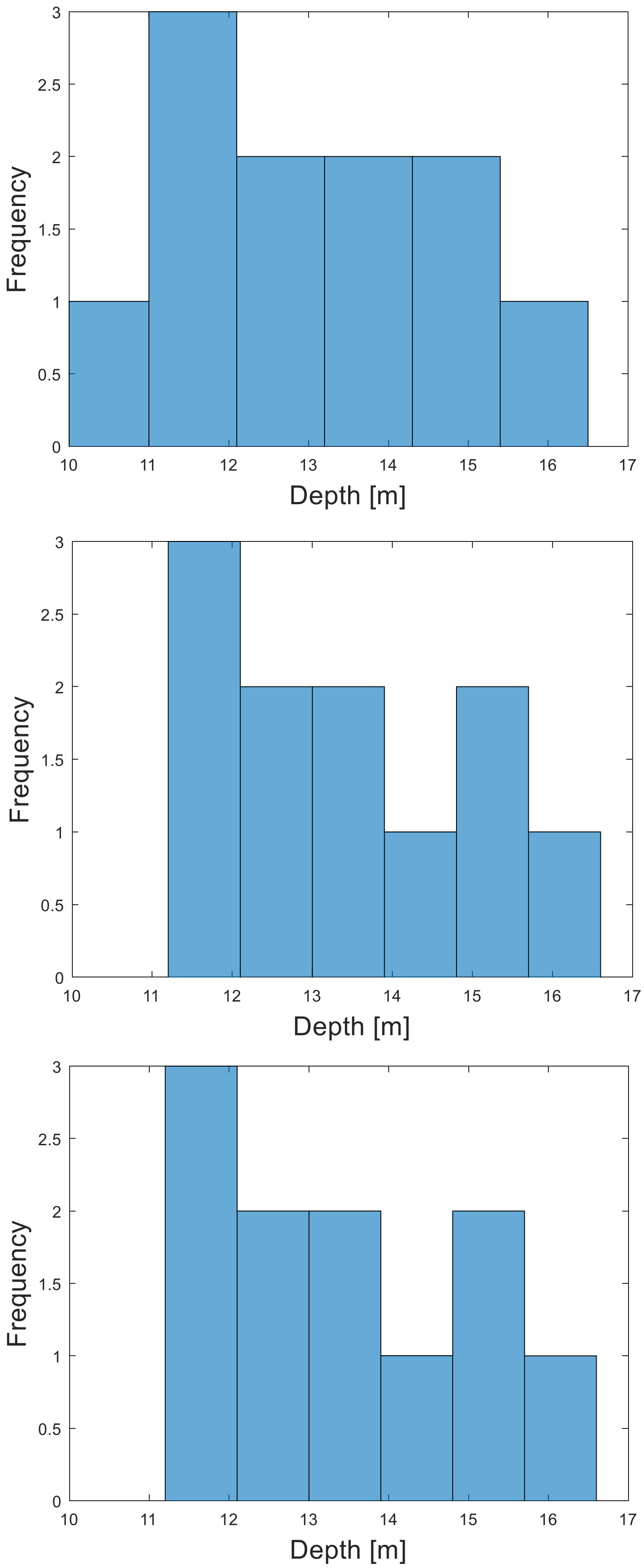


Figure 5. Synthetic model consisting of two homogeneous velocity layers by (*a*) Plus-minus method (*b*) Conventional reciprocal method.

(a)



(b)

(c)

Figure 6. Histogram of estimated depths by the two methods in comparison with the true model (*a*) Original data (*b*) plus-minus (method 1) (*c*) conventional reciprocal method (Method 2).

3.1. Statistical analysis of the methods

To statistically analyze which method is more efficient than the other, we adopted a randomized complete block design (RCBD) which is one of the basic methods used in experimental design. In our design, the treatments are the model, plus-minus and conventional reciprocal methods while the blocks are the geophone positions.

3.2. Theorem

In general, RCBD design is of the form shown in Table 2 (Montgomery, 2013)

Table 2. Theory of RCBD.

	T_1	T_2	T_3	.	.	T_i
B_1	y_{11}	y_{21}	y_{31}	.	.	y_{ij}
B_2	y_{12}	y_{22}	y_{32}	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
B_b	y_{1b}	y_{2b}	y_{3b}	.	.	y_{ib}

Table 2 shows the treatments $(T_{1,}T_2 \dots \dots \dots \dots \dots \dots \dots . T_i)$ against blocks $(B_1, B_2 \dots \dots \dots \dots \dots \dots \dots \dots . B_b)$. As mentioned above, the treatments are the model (plus-minus and conventional reciprocal methods) while the blocks are the geophone positions. Each response y_{ib} represent the depth from method i geophone location b.

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \dots \dots \dots \dots \dots \dots \dots \dots 13$$

where y_{ib} = depth from method i geophone location b

$\mu =$ Overall average depth (mean effect)

$\tau_i = (\mu_i - \mu) =$ Real contribution of i^{th} model

$B_b = (b_b - b)$ =Real contribution of b^{th} block (geophone position)

ε_{ib} = Variation due to experimental error.

Using the least square error approximation:

$$\sum_{i}^{1}\sum_{j}^{b}\varepsilon_{ij}^{2} = \sum_{i}^{1}\sum_{j}^{b}(y_{ij} - \mu - \tau_i - \beta_j)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .14$$

Taking derivatives with respect to μ, τi, βj; and since we have 't' and 'b' linearly independent equations for both treatment and block with one redundant equation for each, then imposing the constraint that the sum of deviation from mean is zero, i.e.

$$\sum_{i}^{1}\tau_i = 0;\ \sum_{j}^{b}\beta_j = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 15$$

We obtained

$$\hat{\mu} = \bar{y}_{..};\ \tau_i = \overline{y_{i.}} - \hat{\mu};\ \beta_j = \overline{y_{.j}} - \hat{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .16$$

For experiment involving RCBD, we tested the equality of the treatment means i.e.

$$H_o: \mu_1 = \mu_2 = \cdots \mu_t;\quad H_1: At\ least\ one\ \mu_i \neq \mu_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots ..17$$

Note for RCBD,

yi. = sum of all observations taken under treatment i;

y.j = sum of all observations in block j;

y.. = grand total of all observations;

N(tb) = total number of observations;

$$\overline{y_{i.}} = \frac{y_i}{b} = \text{average of observations taken under treatment i}$$

$$\overline{y_{.j}} = \frac{y_{.}}{i} = \text{average of observations in block j}$$

$$\overline{y_{..}} = \frac{y_{..}}{N} = \text{grand average of all observations}$$

The total corrected sum of squares is given as:

$$\sum_{i=1}^{t}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{..})^2$$

$$= b\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2$$

$$+ t\sum_{j=1}^{b}(\bar{y}_{.j} - \bar{y}_{..})^2 + \sum_{i=1}^{t}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \ldots\ldots\ldots\ldots\ldots.18$$

Thus in general the partition of the total sum of squares for RCBD is:

$$SS_T = SS_{treatments} + SS_{blocks} + SS_E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..19$$

where:

SS_T= treatment sum of square

SS_{blocks}=block sum of square

SS_E=error sum of square

Since there are N observations, SS_T has N-1 degrees of freedom. There are t treatments and b blocks, so $SS_{treatments}$ and SS_{blocks} have t-1 and b-1 degrees of freedom respectively.

The error sum of squares is the sum of squares between cells minus the sum of squares of treatments and blocks. There are tb cells with tb-1 degree of freedom between them. So SS_E has tb-1-(t-1)-(b-1) = (t-1)(b-1) degrees of freedom. Finding the expected values of SS_T partitions divided by their degree of freedoms resulted into:

$$E(MS_{treatments}) = \sigma^2 + b\frac{\sum_{i=1}\tau_i^2}{t-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..20$$

$$E(MS_{blocks}) = \sigma^2 + t\frac{\sum_{j=1}\beta_j^2}{b-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..21$$

$$E(MS_E) = \sigma^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..22$$

where:

$E(MS_{treatments})$=expected value of treatment mean square

$E(MS_{blocks})$=expected value of block mean square

$E(MS_E)$=expected value of error mean square

Thus to test the equality of the treatment means, the test statistic is

$$F_o = \frac{MS_t}{MS_E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 23$$

which is distributed as $F_{t-1, (t-1)(b-1)}$ if the null hypothesis is true. The critical region is the upper tail of the F distribution, and we reject H_o if $F_o > F_{\alpha, t-1, (t-1)(b-1)}$.

To this end, we performed the RCBD experiment using the 'minitab' statistical package and the data is normalized by taking the inverse.

However, prior to testing as shown in Fig. 7, we check the graphical model so as to validate the normality of our data. From the graphical model, the plots of residuals against fits show no apparent patterns to indicate either interaction between the treatments (model and the two methods) or the position of the geophone. This shows that the residual has a constant variance.

The normal probability plot is consistent with the normality of error with a p-value of 0.319, thereby confirming the normality of the data. Residual versus observation order shows the independent relationship (uncorrelation) between the treatments output and lastly the histogram shows no outliers in the data.

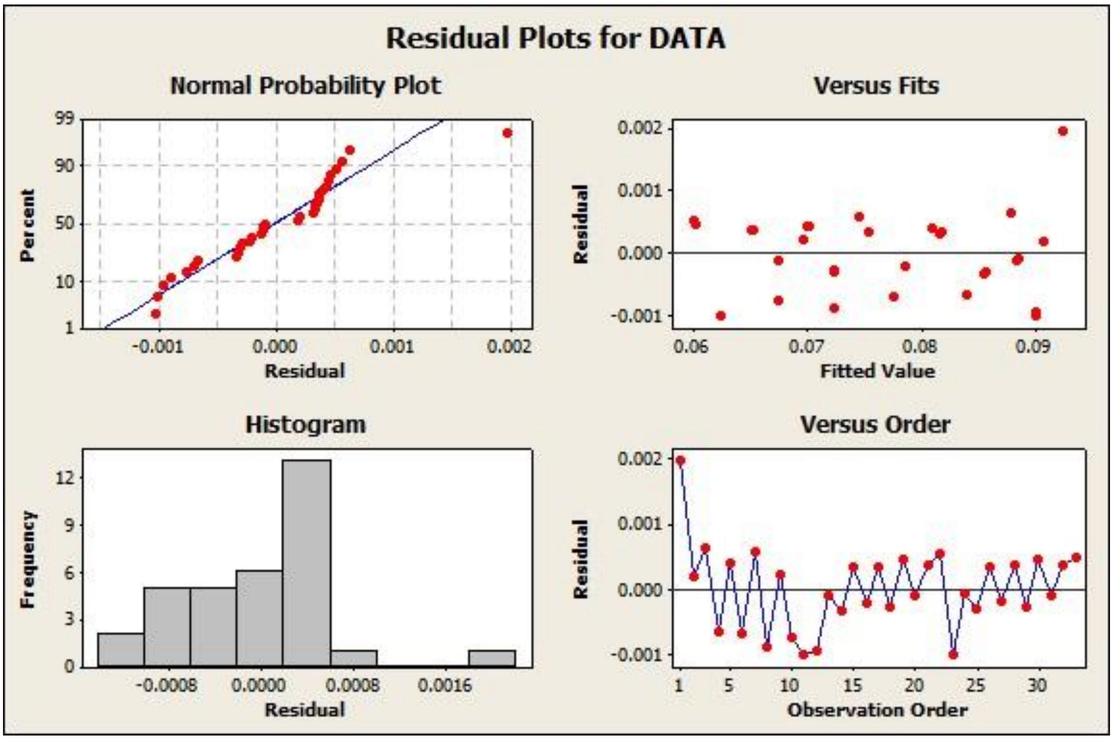


Figure 7. Residual plots.

Table 3 represents the output of the design, since we are interested in testing the null hypothesis of no difference between treatment means against the alternative hypothesis of difference between two means; we consider the p-value for the treatments (method) as shown in the table. The p-value 0.000 suggest that the type of treatment (model) has a significant effect on the response obtained and thus the null hypothesis of no difference should be rejected.

Table 3. Two-way ANOVA: DATA versus METHOD, LOCATION.

Source	DF	SS	MS	F	P
Method	2	0.0000373	0.0000186	30.46	0.000
Location	10	0.0029362	0.0002936	480.22	0.000
Error	20	0.0000122	0.0000006		
Total	32				

To solve this problem so as to ascertain or affirm the best out of the two methods (i.e. plus-minus and conventional reciprocal methods) that best imitate the original model, we proceed to form a confidence interval for differences in the effect of treatments. The Tukey msd for RCBD design is computed as follows:

$$MSD = W_T * \sqrt{MSE * \frac{n_1 + n_2}{n_1 n_2}} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. 24$$

$$W_T = \frac{t_{\alpha,2n-2}}{\sqrt{2}} \dots \dots \dots \dots \dots \dots \dots 25$$

$$W_T = \frac{2.086}{\sqrt{2}} = 1.475 * 0.00033029 = 0.0004871 \dots \dots \dots \dots \dots \dots 26$$

Using descriptive statistics to compute $\bar{y}_{i.}$ for the treatments, where i = 1, 2, 3 yielded 0.078169, 0.07589 and 0.075942 respectively. The centers of the simultaneous confidence interval for the pairwise difference contrast is given in Table 4.

Table 4. Center of Confidence Intervals for the pairwise difference contrast.

Contrast	Center of CI
$\tau_1 - \tau_2$	0.0022794
$\tau_1 - \tau_3$	0.00222732
$\tau_2 - \tau_3$	-5.20763E -05

If the result is compared with MSD, then it is obvious that the two methods provide a good estimation of the original model. However, the pairwise comparison between the two methods suggests that plus-minus method is better than the conventional reciprocal method.

4. Conclusion

The results of plus-minus and conventional reciprocal methods have been compared qualitatively and quantitatively using a synthetic data acquired from a slightly dipping refractor. The models obtained from both methods provide a close approximation to the actual geophysical model. Geophysical parameters such as velocities, dips and depths beneath each geophone are reasonably comparable to the actual measurements which suggests that both methods can be employed in shallow refraction study of the near surface.

Further statistical test using randomized complete block design suggests that the method type is significant, and this necessitate the need for further test. Using pairwise comparison, the results suggest that the two methods provide a good representation of the actual model, however the result generated by plus-minus method better imitates the actual geophysical model than the conventional reciprocal method.

We recommend testing the methods on complex model to properly evaluate this claim.

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