GEOFIZIKA	Vol. 3	1986

Original scientific paper
UDC 551.513

One-dimensional spectral analysis of the eddy available potential energy growth over a limited region

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Received 21 July, 1986, in final form 25 November, 1986.

The paper describes development processes on their energetic basis. A special emphasis is given to the role of the eddy available potential energy, A_e arising out of zonal available potential energy, A_z. The process of energy transition, $C(A_z A_e)$ can be estimated in the wave number domain by means of a cospectral function $\phi_n(H, T)$ of the isobaric surface height H and the temperature T. In cases of baroclinic instability prevailance, the $\phi_n(H, T)$, if determined along a cross-sectional line over a limited region, may help the forecast of a given eddy scale development, relevant for that region.

Jednodimenzionalna spektralna analiza razvoja raspoložive potencijalne energije perturbacija iznad ograničenog područja

Članak opisuje procese razvoja na njihovoj energetskoj osnovi. Specijalan naglasak je dan nastajanju raspoložive potencijalne energije perturbacija, A_e iz zonalne raspoložive potencijalne energije, A_z. Proces prijelaza energije, $C(A_z A_e)$ može se procijeniti u oblasti valnog broja pomoću kospektralne funkcije $\phi_n(H, T)$ visine izobarne plohe H i temperature T. Ako je u slučaju prevladavanja barokline nestabilnosti $\phi_n(H, T)$ određena duž linije koja presijeca ograničeno područje, ona može pomoći u prognoziranju razvoja poremećenja određene veličine, relevantne za to područje.

1. Introduction

Since Lorenz (1955) introduced to the atmosphere the old idea of available potential energy (Margules, 1903), the energetics of atmospheric macroscale dynamics has been studied extensively. The energy cycle of the atmosphere general circulation consists of generation, transition, transformation and dissipation of the mean and eddy energies. Due to the eddy scales variability a spectral technique approach has become a common tool for the atmospheric energetics study (van Mieghem, 1952; Saltzman, 1957, 1970 etc; Tsing-Chang Chen and Marshall, 1984, etc.). The decomposition of the energy components in the wave number domain helped the better understanding of our planetary circulation. It helped the discovery of the enstrophy conservation in the twodimensional turbulence field of a quasigeostrophic atmosphere connected with a two-directional energy flow out of the most energetic cyclone perturbations (Fjortoft, 1953). Planetary considerations give insight into a possible development of general circulation regimes and can improve also the understanding of atmospheric energetics over a specific region. Obviously, regional atmospheric energetics is closely related to the planetary one and cannot be "extracted" out of the planetary circulation (Smith, 1969). Still, global energy cycle investigations, particularly when treating transient eddies statistics (Ngar-Cheung Lau and Oort, 1982; Arpe, Brankovic and Oriol, 1986) detect some zones and regions with more or less characteristic energy behaviour. In other words, one can try to investigate the energy cycle components over a given region being a part of the planetary circulation energy cycle (Edmon, 1978).

This paper describes such an attempt which comprises only a study of the conversion from zonal to eddy available potential energies over a given region due to the horizontal eddy flux of heat over the region. It is based on one-dimensional spectral analysis of the geopotential height H and the temperature T fields on the given isobaric surface. Cospectra of H and T can help to extract solely a transition from zonal to eddy available potential energy out of the whole energy cycle. In case of the eddy available potential energy A_e dominance in the energy cycle (a baroclinically unstable air current) a cospectrum function $\phi_n(H,T)$ indicates a possible development of specific eddy scales over the region.

2. Cospectrum energy function

The total energy of an eddy, E_e, can be described as a sum of the available potential, A_e and the kinetic, K_e eddy energies, i. e.

$$E_e = A_e + K_e \tag{1}$$

A change of this energy consists of the available potential eddy energy variation due to an energy transition from zonal to its eddy form, $C(A_z A_e)$, then of the transformation of eddy available potential to eddy kinetic energy, $C(A_e K_e)$, further of the kinetic energy transitions amongst eddies of various scales including the zonal flow, $C(K_e K_z)$ and of the kinetic energy dissipation, $D(K_e)$:

$$\frac{dE_e}{dt} = C(A_zA_e) + C(A_eK_e) + C(K_eK_z) + D(K_e) \quad (2)$$

where subscripts "z" and "e" denote "zonal" and "eddy" values respectively.

An energy transition $C(A_zA_e)$ is due to the meridional transport of the perturbation temperature by perturbation winds, $\overline{v'T'}$ in a baroclinically unstable zonal current. Simultaneously, this eddy available potential energy is being transformed into its kinetic form, $C(A_eK_e)$ by an overturning with the warm air rising and the cold air sinking, $\overline{\omega'T'}$. Still the eddy available potential energy can further grow provided it receives from A_z more than it gives to K_e i.e. in cases when the horizontal eddy temperature transport $\overline{v'T'}$ exceeds the vertical one $\overline{\omega'T'}$ in

$$\frac{dE_e}{dt} \propto \overline{v'T'} + \overline{\omega'T'} + C(K_eK_z) + D(K_e) \quad (3)$$

Here "——" denotes the zonal average, v' is the meridional velocity perturbation, T' temperature perturbation and ω' the vertical velocity perturbation in isobaric coordinates. The equation (3) is fairy simplified version of the eddy energy variation and we are going to determine only its first term or A_e development term

$$C(A_zA_e) \propto \overline{v'T'} \quad (4)$$

in the wave number domain and over a given region.

A positive value of $\overline{v'T'}$ denotes an energy transition from zonal to eddy forms. Since v' in a quasigeostrophic atmosphere above the planetary boundary layer is directly correlated to the height perturbation H' it follows that the mutual position of H and T fields may indicate the possible sign of $C(A_z, A_e)$.

In cases of $C(A_z, A_e) > 0$ the energy "flows" to perturbations and they amplify. The air current has a baroclinic instability. This is the well known case of temperature wave lags behind a height wave.

Still, the isobaric surface heights and corresponding wind fields comprise the resulting influence of all four terms in (2). Therefore, a decrease of E_e in a baroclinic atmosphere usually indicates that more eddy energy has been transformed from A_e to K_e than it is generated by the transition from A_z to A_e. In such a case the perturbation amplitude attenuates but it may happen that winds strenghten at the same time (provided that above PBL the dissipation term is usually less than other terms containing K_e).

In our approach we cannot separate $\overline{v'T'}$ and $\overline{\omega'T'}$ mechanisms since they are "working" simultaneously but we can, at least, judge on $C(A_zA_e)$ by $\overline{v'T'}$ alone. Also, some barotropic energy transitions are present due to the momentum transport. This effect is taken to be of minor importance in cases of baroclinic instability. If on the contrary, the barotropic instability appears to be a dominant one, calculations of $\overline{v'T'}$ will fail to predict the development. This again is not incompatible with this paper purpose to estimate $C(A_zA_e)$ alone.

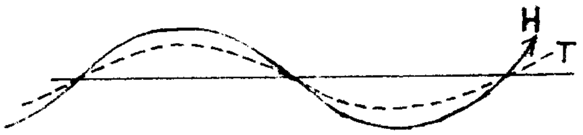
The wave-like macroscale disturbances may appear very irregular in their shapes and proportions so that one might come to wrong conclusions about the H and T mutual positions. Therefore, a decomposition of these fields into their harmonic components with regular shapes and with known phase lags makes it possible not only to conclude on H and T mutual position but also to separate those wave components which are mostly unstable. A very simple mathematical description by means of cospectra technic serves the purpose. Similarly to van Mieghem's way of forming temperature and vertical velocity cospectra (van Mieghem,1960) we define here the function

$$\Phi_n(H,T) = aA_n(H)B_n(T)\sin[\alpha_n(H)-\beta_n(T)] \tag{4}$$

which describes the energy transition $C(A_z,A_e)$ in the wave number domain. Here $A_n(H)$, $B_n(T)$, $\alpha_n(H)$ and $\beta_n(T)$ represent height and temperature waves amplitudes and phase lags respectively. Proportionality constant "a" can be taken 1 for the purpose of this paper.

The efficiency of the eddy energy cospectral function $\Phi_n(H,T)$ is illustrated by the following scheme:

(a)



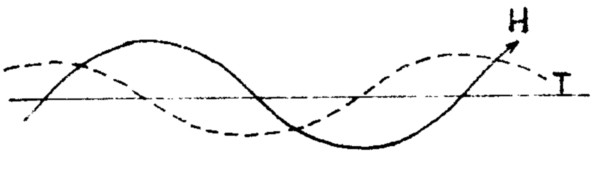
$\Phi_n(H,T) = 0$

since $\alpha_n(H) - \beta_n(T) = 0$

$C(A_z,A_e) = \overline{v'T'} = 0$

The perturbation is in a "neutral" state. It is not expected to amplify.

(b)



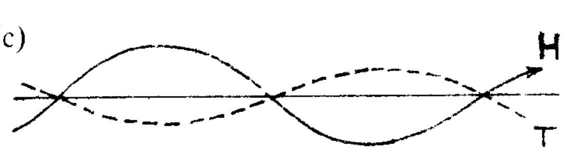
$\alpha_n(H) - \beta_n(T) = 90^\circ$

$\Phi_n(H,T) > 0$

$C(A_z,A_e) > 0$

The perturbation in the height field will grow. It is baroclinically unstable.

(c)



$\alpha_n(H) - \beta_n(T) = 180^\circ$

$\Phi_n(H,T) = 0$

$C(A_z,A_e) = 0$ "Neutral" case.

Follows: the given perturbation of wave number n is baroclinically unstable in case of

$$0^\circ < \alpha_n(H) - \beta_n(T) < 180^\circ$$

with a maximum instability at $\alpha_n(H) - \beta_n(T) = 90^\circ$. The wave amplifies.

(d)

$\alpha_n(H) - \beta_n(T) = 270^\circ$

$\Phi_n(H,T) < 0$

$C(A_z, A_e) < 0$

The perturbation is baroclinically stable.

Follows: the given perturbation of wave number n is stable in the case of

$$180^\circ < \alpha_n(H) - \beta_n(T) < 360^\circ$$

with a maximum stability at $\alpha_n(H) - \beta_n(T) = 270^\circ$. The wave is damped or it does not change its amplitude.

3. One-dimensional spectral transformation

Estimations of the energy transition spectral function $\Phi_n(H,T)$ by a spectral analysis of H and T fields along given cross sectional lines on the given isobaric surface can be easily performed if taken all over the Earth (usually along a latitude circle – Oort, 1964). Unfortunately, these results are almost impossible to be applied to a given smaller region since they are equally valid all around the circle. Therefore one may try to perform the spectral analysis along the lines which cross only the specified given region. The boundary problems, which naturally arise, and the influence of wave disturbances outside the cross section upon those inside it (Barret, 1961) are supposed to be overcome by prolonging the cross-sectional line reasonably enough at both sides to make sure that results are mostly applicable in its middle part. The main drawback of such an approach is in the fact that a choice of the position, the length and the shape of cross-sectional lines are at present subjective. The greater number of lines the more reliable the results. An objective method of cross-section choice is still required.

Generally, the cospectral function $\Phi_n(H,T)$ indicates not only the possible development (attenuation) but – more important – *it indicates the possible scale of baroclinically unstable eddies.*

It should be mentioned here that in spite of some investigations of atmospheric energy processes duration (e.g. Rosen and Salstein, 1982) it is still a research subject to detect the time interval relevant to $\Phi_n(H,T)$. Therefore, the values of $\Phi_n(H,T)$ in Tables 1, 2 and 3 have only indicative meanings.

The role of other terms in the equation (2) can be evaluated by means of other fields cospectral functions. This is our next task.

4. Method illustrations

Three examples of the energy transition estimation by means of $\Phi_n(H,T)$ are presented, all of them making use of 500 hPa isobaric surface heights H and corresponding temperature data, based on 00^h GMT soundings. The region under consideration is the middle part of Europe on 13, 19 and 20 December, 1957 (Figs. 1, 2, 3, 4 and 5). Fourier series of H and T converge relatively fast in all cases so that only the first six harmonics were enough to describe the expected energy transition $C(A_z, A_e)$ and the

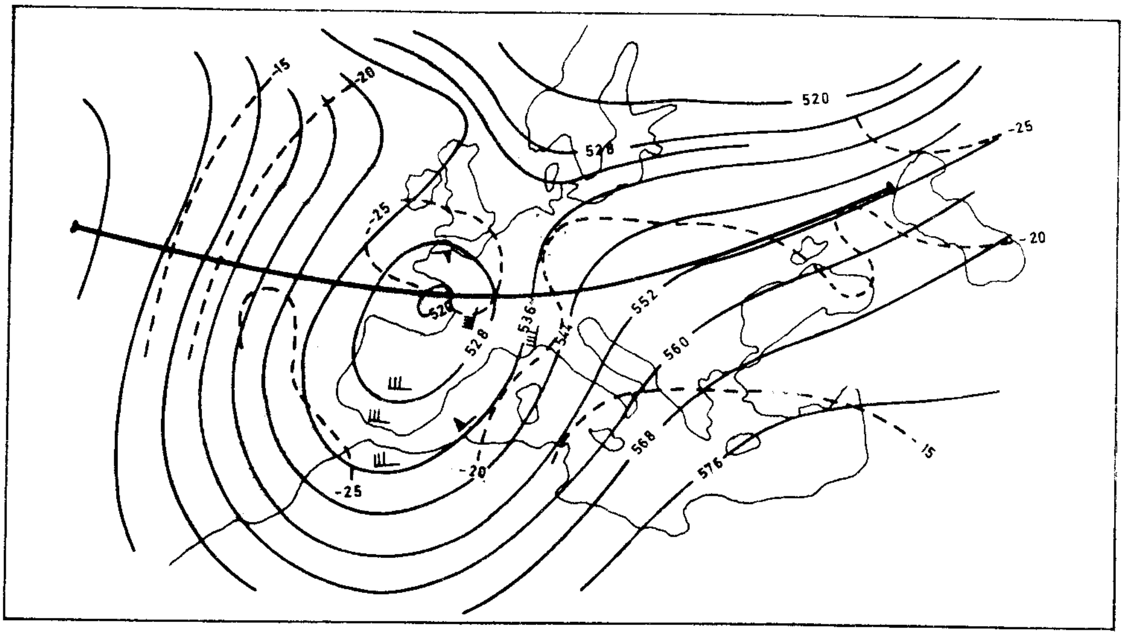


Figure 1. A draft of AT 500 hPa, 00 GMT, 13 December 1957 (based on Täglicher Wetterbericht), . . . isotherm, °C

resulting baroclinic instability of the given wave number (or wave length – see Tables 1, 2 and 3). The cross-sectional lines are denoted on all figures by a thick line. Having

Table 1. Energy transition function $\Phi_n(H,T)$

December 13, 1957 – AT 500hPa, 00^h GMT

wave number, n	1	2	3	4	5	6
$A_n(H)$, gpm	210,8	98,6	41,5	23,0	25,8	21,5
$B_n(T)$ °C	4,7	3,2	1,0	1,2	0,9	0,7
$\alpha_n\ (H)^\circ$	72	325	41	357	35	38
$\beta_n\ (T)^\circ$	97	349	318	276	37	359
$\Phi_n(H,T)$	-412,7	-128,5	42,6	27,0	-0,7	6,6
L, km	7300	3650	2433	1825	1460	1217

synoptic charts only at 00^h we could test the results of predicted energy transition only after 24 hours. Macro-scale experiments often make use of such or greater intervals.

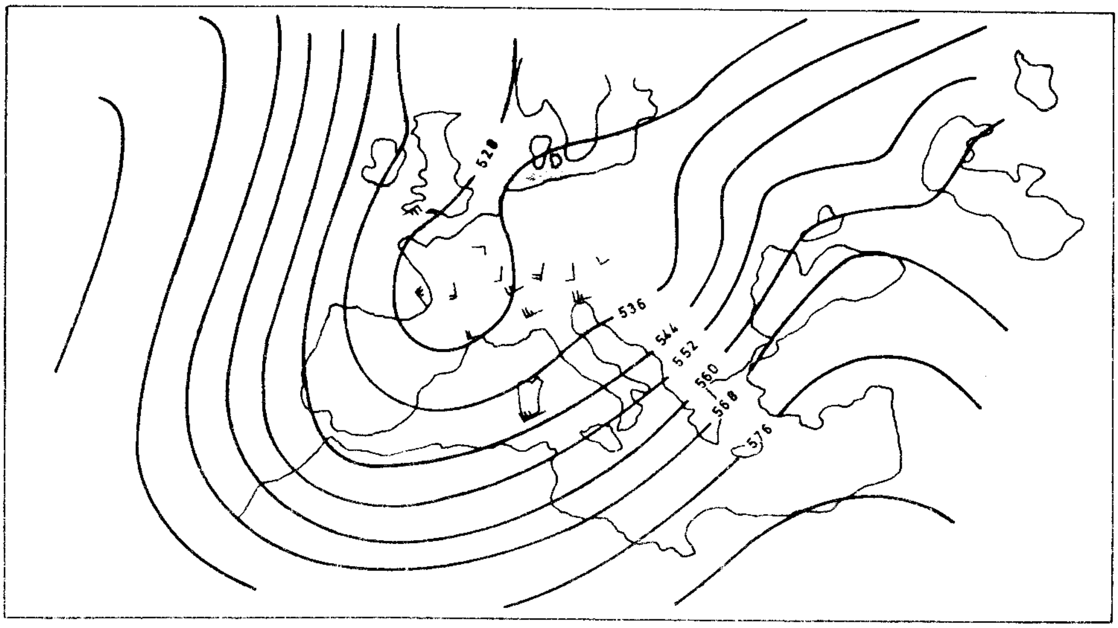


Figure 2. A draft of AT 500 hPa, 00 GMT, 14 December 1957 (based on Täglicher Wetterbericht 1957)

Table 2. Energy transition function $\Phi_n(H,T)$

December 19, 1957, 00^h – AT 500hPa

wave number, n	1	2	3	4	5	6
$A_n(H)$, gpm	80,7	8,2	9,7	6,9	0,5	3,3
$B_n(T)$, °C	3,4	2,7	0,8	1,5	0,0	0,9
$\alpha_n(H)^\circ$	70	57	59	59	350	90
$\beta_n(T)^\circ$	130	214	166	243	0	90
$\Phi_n(H,T)$	-237,6	-8,6	-7,5	0,1	0,0	0,0

According to large negative values of $\Phi_n(H,T)$ for $n=1$ and 2 we expect the big cyclone over Biscai to weaken. Weakening of a depression presents the case when $C(A_e K_e)$ and $C(K_e K_z)$ are stronger than $C(A_z A_e)$. In other words, negative values of $\Phi_n(H,T)$ indicate that the perturbation receives less potential energy from the mean flow than it transforms to its kinetic energy (and further, but not necessarily, back to the kinetic energy of the mean flow). The next day the perturbation over Biscai attenuated and the wind field got rather "noisy" appearance.

Table 3. Energy transition function $\Phi_n(H, T)$

December 20, 1957, 00^h *– AT 500hPa*

ware number, n	1	2	3	4	5	6
$A_n(H)$, gpm	140,2	141,2	56,8	22,3	30,4	14,1
$B_n(T)$, °C	5,8	1,9	2,1	0,9	1,2	0,2
$\alpha_n(H)°$	61	58	339	93	24	0
$\beta_n(T)°$	278	103	319	152	163	349
$\Phi_n(H,T)$	487,9	-187,1	35,8	-18,1	25,4	0,6

$\Phi_n(H,T)$ series on 19 December 1957 indicate that the greatest perturbation connected with the wave number 1 might weaken but failed to predict explicitely the development of the wave number 2 the next day. This is an example of the weakened

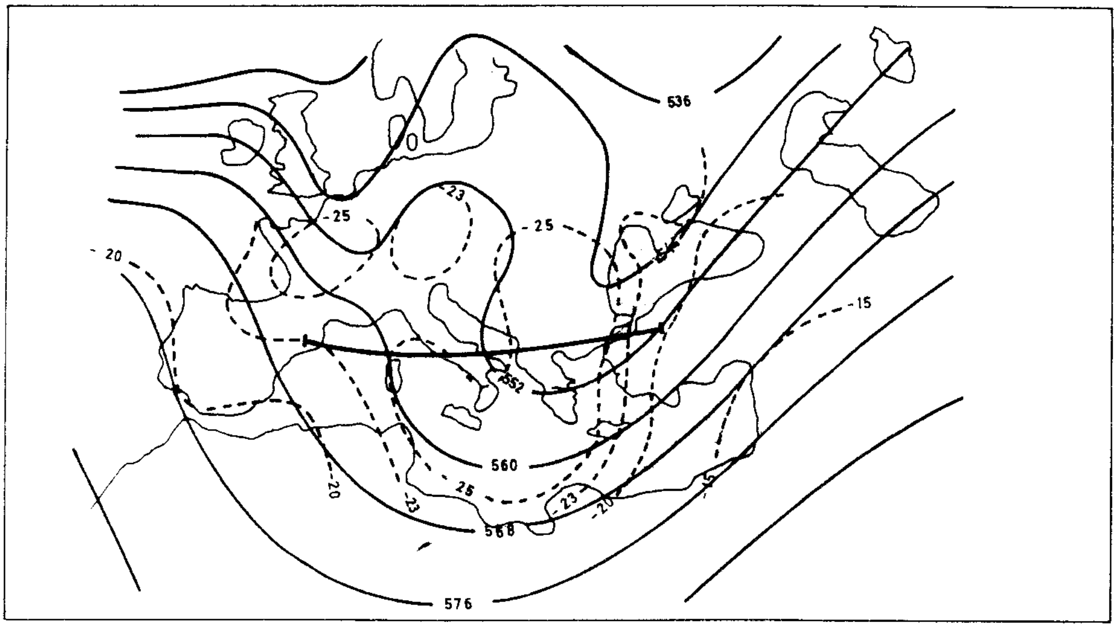


Figure 3. A draft of AT 500 hPa, 00 GMT, 19 December 1957 (based on Täglicher Wetterbericht 1957), . . . isotherm, °C

baroclinic instability. In such a case barotropic energy redistribution amongst eddies of various wave numbers may become important and the $\omega'T'$ mechanism of $C(A_e K_e)$ as well. Obviously, the $\Phi_n(H, T)$ method of various eddy scales development prediction can be efficient only when the baroclinic instability prevails.

According to Tab. 3 the perturbation of the wave number $n=2$ at 20 December might weaken and, even more instructive, a wave with $n=1$ might develop. In other words, two AT 500 hPa lows at 20 December are not expected to deepen separately but to connect in an extended macroperturbation of the wave number 1. The prediction was confirmed the following day.

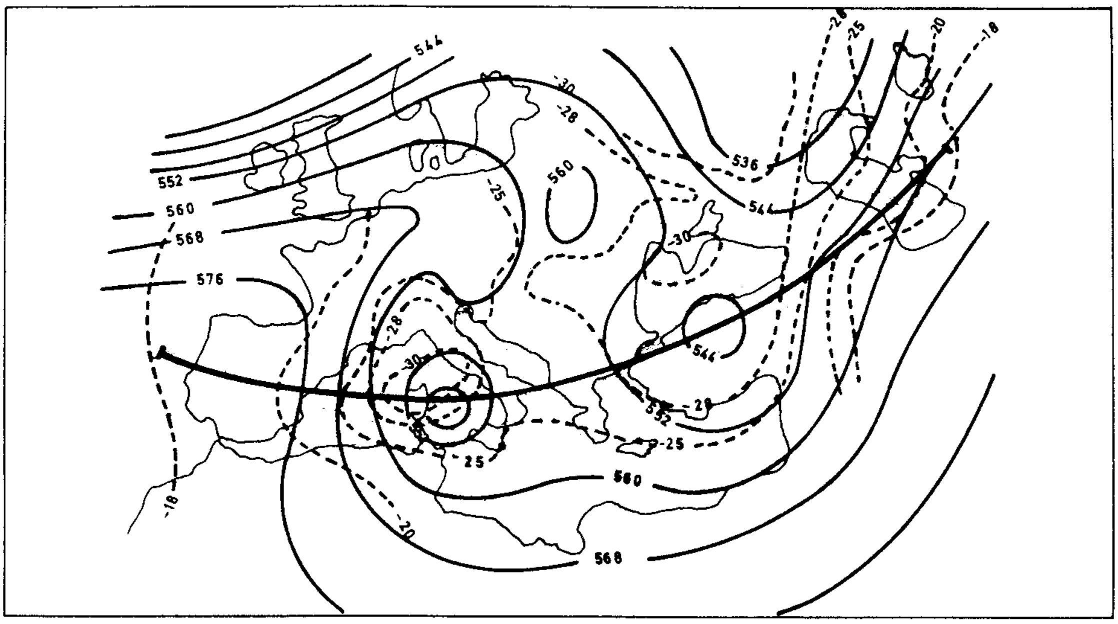


Figure 4. A draft of AT 500 hPa, 00 GMT, 20 December 1957 (based on Täglicher Wetterbericht 1957), . . . isotherm ° C

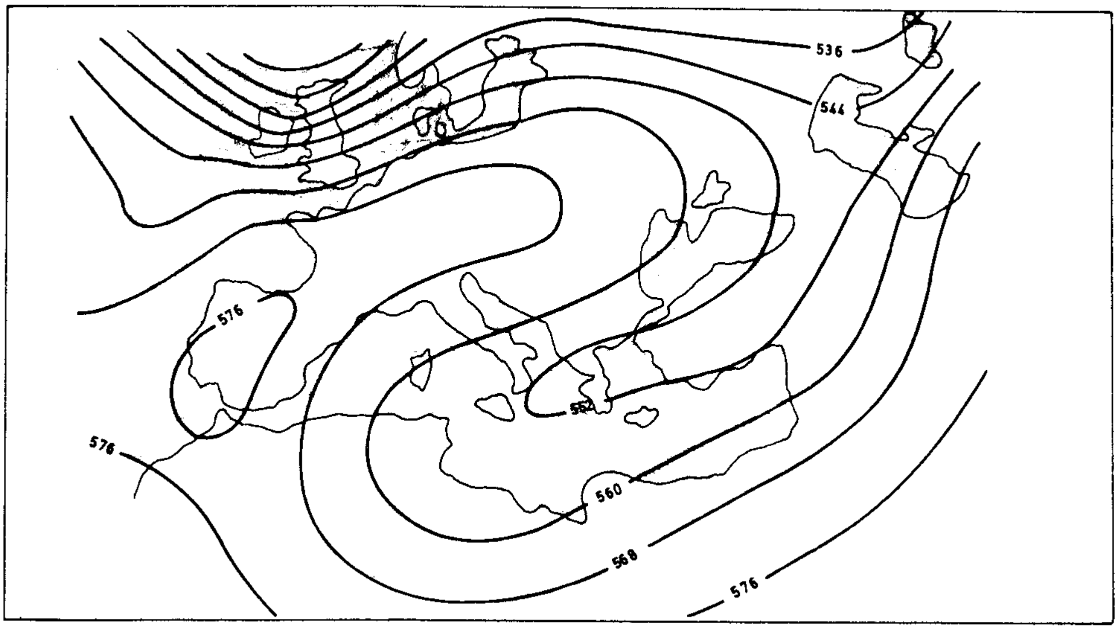


Figure 5. A draft of AT 500 hPa, 00 GMT, 21 December 1957 (based on Täglicher Wetterbericht 1957)

5. Conclusion

Several examples shown in the previous section give some confirmation of the $\Phi_n(H, T)$ function practical usefulness, particularly in cases of $\Phi_n(H, T) > 0$. Besides, such a simple approach helps to make clear that a transition of available potential energy from zonal (mean) to the eddy type, $C(A_z\, A_e)$ should not be mixed with the subsequent (or simultaneous) transformation from an eddy available potential to the eddy kinetic energy $C(A_e\, K_e)$, which takes place by a different mechanism. In other words, an eddy energy generation connected with a baroclinic instability of the solenoidal atmosphere may cause the deepening of the height perturbation field. This can but need not necesarily be followed by a further energy transformation to its kinetic form. The sum of $C(A_z\, A_e)$ and $C(A_e K_e)$ may even tend to stay conservative, so the air flow becoming more perturbed may even be accompanied by the height perturbation weakening (as it was the case on 13/14 December 1957). However, these processes usually go jointly in the nature and it is not always clear which one is stronger, particularly if taking into account a simultaneous kinetic energy redistribution amongst various perturbation scales due to barotropic instability and connected with an eddy momentum transport (which is not considered in this paper).

The proposed method of "extracting" the $C(A_z\, A_e)$ influence alone out of the whole cycle of energy conversion has an advantage in its ability to predict the scale of a possible disturbance baroclinic development relevant for the given region. Also a testing of altogether 20 cases has indicated that the energy transition $\Phi_n(H, T)$ might have an approximate duration of 24 hours.

The method still has to be proved by much more experiments which should satisfy appropriate statistical verification tests.

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