

## Study of statistical probability of solar irradiation energy levels in Zagreb and their application

*Ivan Penzar and Mario Žic*  
*Geophysical Institute, Zagreb,*  
*and Electrotechnical Institute, Rade Končar, Zagreb*

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In order to obtain a more accurate evaluation of efficiency of the solar system the measurements of global irradiation by Moll-Gorczynski solarigraph were analysed with respect to levels of energy instead to arithmetic means of irradiation. The time variation of irradiation levels in daily and yearly course is considered. The structure of global irradiation energy presented on the tilted plane and put in the mathematical model improved the simulation of work of the entire solar installation.

### Proučavanje statističke vjerojatnosti energetskeg pragova Sunčeva zračenja u Zagrebu i njihova primjena

Za dobivanje ispravnije procjene efikasnosti rada solarnog sistema, umjesto aritmetičkih srednjaka dozračene energije, analizirani su ovdje energetskeg pragovi globalnog zračenja izmjerenog solarigrafom Moll-Gorczynski. Razmatra se dnevni i godišnji hod energetskeg pragova. Podaci o energetskeg strukturi dozračene Sunčeve energije preračunati su na kosu plohu i poslužili su u matematičkom modelu za simulaciju rada cjelokupnog solarnog postrojenja.

### 1. Introduction

Standard statistical treatment of global solar irradiation as usually done in meteorological practice is often inadequate for solar application. For this purpose it is more useful to know the structure of energy i. e. the share of low and high solar irradiation within given time intervals than to know ordinary mean values. This paper presents the method and an example of such data processing.

As the collectors are inclined in relation to the horizon, we want to calculate irradiated energy on a tilted plane from measurements done on a horizontal plane. Total irradiation should therefore be divided into direct and diffuse component. Tilted planes receive diffuse irradiation only from the part of the sky they face.

The use of mean irradiation values leads to erroneous results, the error being greater, the longer is the time period, but the error decreases if, instead of mean values, real, cumulative global irradiation hourly frequencies are used in order to derive percentual shares of given energy amounts.

## 2. Data and method

The hourly global irradiation values measured by the pyranometer at the Zagreb Grič observatory over 15 years (1968–1982) were used. Every hour between the sunrise and the sunset yielded 420 to 465 data for analysis, depending on the number of days in a month. For each hour the frequency distribution was made. The number of the classes ranged from 10 to 20 according to the dispersion of the data.

The interval 0 to 100 for the relative cumulative frequency was divided into 10 equal parts and in the middle of each part (i. e. at points  $f = 5, 15, 25, \dots, 95$ ) the corresponding energy  $W(f)$  of global irradiation was interpolated by means of the cubic interpolation spline with numerically evaluated boundary conditions:

$$W(f) = W_i + \alpha_i(f-f_i) + \beta_i(f-f_i)^2 + \gamma_i(f-f_i)^3 \quad \text{for } f_i \leq f \leq f_{i+1}$$

The spline was given by the interpolation points  $f_i$  ( $i = 0, 1, \dots, n$ ,  $n$  ranging between 10 and 20) and the corresponding energies  $W_i$ .

## 3. Thresholds of global irradiation

The procedure divided the global irradiation energy within each hour into ten parts. For the further research the supposition that each of these parts belongs to one-tenth of an hour, i. e. to a 6-minute interval, was necessary. In order to shorten the conversion from a horizontal to a tilted plane, the mean values from the corresponding morning and afternoon hours, presented in Tab. 1, were used.

The energy thresholds exceeded with the probability of 10, 50, and 90 percent are shown in Fig. 1, 2, and 3 respectively. High thresholds with a 10 percent probability (Fig. 1.) amount to over  $900 \text{ Wm}^{-2}$  in the summer and to  $300 \text{ Wm}^{-2}$  in the winter about noon. In the summer at midday the 50 percent probability belongs to the energy larger than  $700 \text{ Wm}^{-2}$ . In other words, the irradiated energy of  $700 \text{ Wm}^{-2}$  lasts up to thirty minutes. In the winter about midday, the corresponding energy is only  $90 \text{ Wm}^{-2}$  (Fig. 2.). With the probability 0,90 (Fig. 3.) the irradiation exceeds  $140 \text{ Wm}^{-2}$  at noon in summer and only  $10 \text{ Wm}^{-2}$  at noon in winter. One can say also that irradiation of  $140 \text{ Wm}^{-2}$  about summer noon as well as the irradiation of  $10 \text{ Wm}^{-2}$  about winter noon lasts at least 54 minutes.

The average hourly values derived from Tab. 1. are slightly less than the previously known means of global irradiation in Zagreb for the 1958–1967 period (Penzar, 1977).

Table 1. Thresholds of energy  $W(f)$  reached or exceeded by global irradiation in Zagreb with the given probability  $f$  (1968–1982. period)

Probability $f$	HOURS							
	4 - 5 19 - 20	5 - 6 18 - 19	6 - 7 17 - 18	7 - 8 16 - 17	8 - 9 15 - 16	9 - 10 14 - 15	10 - 11 13 - 14	11 - 12 12 - 13
	$W(f)$			$Wm^{-2}$				
	January							
5				37	113	253	344	398
15				26	83	197	289	323
25				17	64	151	229	268
35				12	48	114	173	212
45				9	38	87	132	152
55				8	30	67	98	120
65				7	23	51	76	90
75				6	16	39	62	68
85				5	9	24	46	50
95				2	3	9	19	20
	February							
5		12	87	253	385	496	553	
15		9	58	180	308	421	482	
25		7	45	140	303	361	426	
35		4	36	113	204	307	364	
45		3	28	88	153	223	275	
55		2	21	66	109	163	205	
65		2	15	49	83	126	135	
75		1	9	36	60	89	103	
85		1	5	23	41	59	73	
95		1	2	8	16	28	36	
	March							
5		82	247	425	592	693	756	
15		52	188	355	510	621	678	
25		39	156	307	458	567	623	
35		31	125	261	402	511	565	
45		24	98	222	324	428	494	
55		18	75	171	269	339	382	
65		14	57	130	201	245	277	
75		10	40	86	140	179	209	
85		7	26	58	82	112	135	
95		3	9	26	55	61	81	

Table 1. Continuation a)

Probability $f$	HOURS							
	4 - 5 19 - 20	5 - 6 18 - 19	6 - 7 17 - 18	7 - 8 16 - 17	8 - 9 15 - 16	9 - 10 14 - 15	10 - 11 13 - 14	11 - 12 12 - 13
	$W(f)$				$W m^{-2}$			
	April							
5		48	184	346	554	710	817	844
15		38	152	321	502	673	747	796
25		31	124	288	453	604	699	752
35		25	103	251	409	554	695	689
45		21	91	210	352	500	684	614
55		18	71	175	287	413	492	521
65		15	56	145	229	333	383	402
75		11	41	106	171	253	293	300
85		7	25	69	113	165	192	203
95		3	8	28	44	83	101	110
	May							
5	26	114	302	469	670	791	894	951
15	21	92	246	435	594	748	838	882
25	18	80	232	399	551	693	794	835
35	16	71	208	368	519	659	738	797
45	14	61	184	328	478	614	687	741
55	12	53	155	284	422	548	620	668
65	11	43	122	208	316	419	507	558
75	8	32	72	153	235	300	379	415
85	5	19	50	95	156	190	223	244
95	2	6	22	34	55	91	136	133
	June							
5	33	131	308	492	686	814	897	952
15	25	113	282	439	623	743	839	898
25	18	100	247	408	575	703	797	826
35	16	85	229	375	542	674	747	779
45	15	80	209	344	503	630	695	718
55	13	66	173	304	443	549	620	637
65	9	56	144	230	354	442	513	532
75	6	41	107	189	257	308	358	410
85	3	30	69	117	164	211	259	266
95	1	12	29	48	70	95	137	121

Table 1. Continuation b)

Probability <i>f</i>	HOURS								
	4 - 5 19 - 20	5 - 6 18 - 19	6 - 7 17 - 18	7 - 8 16 - 17	8 - 9 15 - 16	9 - 10 14 - 15	10 - 11 13 - 14	11 - 12 12 - 13	
	<i>W(f)</i>				$W m^{-2}$				
	July								
5	29	126	295	435	657	801	891	942	
15	21	112	265	430	628	747	841	876	
25	14	92	242	419	591	611	804	840	
35	11	84	234	389	538	674	766	810	
45	9	75	209	360	511	639	718	769	
55	7	65	186	332	483	595	660	704	
65	6	54	157	281	427	518	581	607	
75	4	42	116	211	329	446	467	477	
85	2	29	63	137	195	259	285	315	
95	1	11	25	40	87	117	131	134	
	August								
5	17	78	232	428	601	740	836	848	
15	14	56	191	369	529	678	759	777	
25	12	45	177	336	500	635	720	744	
35	10	39	158	311	477	596	697	716	
45	8	33	139	284	446	558	661	679	
55	7	28	122	250	402	515	592	628	
65	5	22	103	204	340	451	525	524	
75	4	16	77	145	256	332	382	385	
85	3	10	50	93	170	211	227	229	
95	1	4	19	37	52	79	104	101	
	September								
5		23	121	305	492	645	753	793	
15		17	96	263	420	562	680	738	
25		13	76	231	377	519	631	686	
35		10	60	208	348	486	587	644	
45		9	49	182	314	448	543	600	
55		7	42	154	267	397	500	550	
65		6	35	114	212	317	414	465	
75		4	26	79	154	232	295	323	
85		2	16	50	93	139	171	184	
95		1	5	18	32	49	63	81	

Table 1. Continuation c)

Probability <i>f</i>	HOURS							
	4 - 5 19 - 20	5 - 6 18 - 19	6 - 7 17 - 18	7 - 8 16 - 17	8 - 9 15 - 16	9 - 10 14 - 15	10 - 11 13 - 14	11 - 12 12 - 13
	<i>W(f)</i>				$W m^{-2}$			
October								
5			31	140	314	485	607	667
15			22	104	255	421	541	590
25			14	82	225	380	492	546
35			10	68	190	347	447	505
45			9	56	162	304	397	450
55			8	46	129	251	333	371
65			7	36	95	199	253	285
75			5	26	70	154	189	212
85			2	16	48	103	143	154
95			1	5	19	37	62	68
November								
5				40	150	288	385	435
15				30	118	245	345	384
25				22	86	210	279	322
35				18	72	161	229	267
45				14	59	120	179	210
55				12	45	89	137	152
65				9	34	71	103	112
75				6	24	55	74	84
85				4	14	36	44	53
95				2	5	13	14	18
December								
5				16	87	209	304	353
15				9	57	175	255	305
25				8	47	132	202	246
35				6	37	96	158	181
45				5	26	69	113	133
55				4	18	53	84	99
65				3	13	43	64	74
75				2	7	33	49	54
85				1	3	20	31	35
95					1	6	11	13

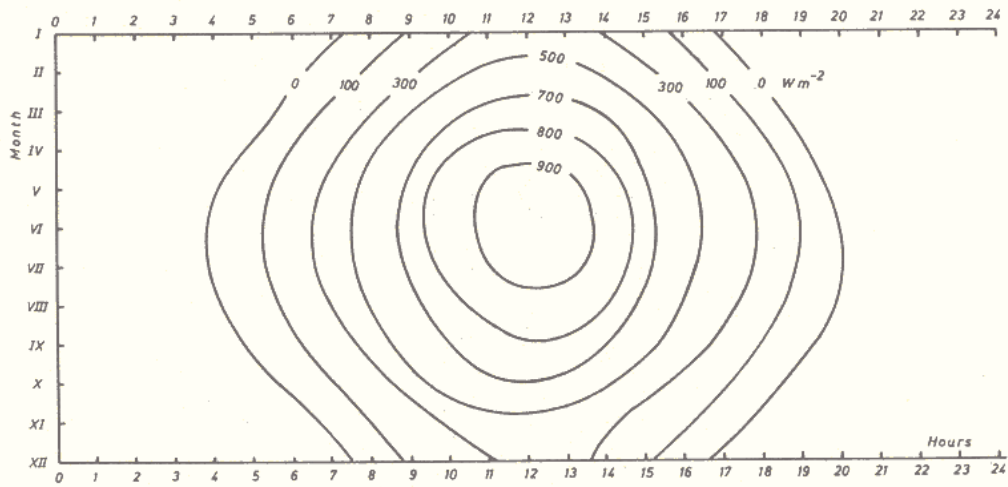


Figure 1. Isopleths of highest energy thresholds reached by global solar irradiation in 10 percent cases, i. e. lasting up to 6 minutes in a single hour of an average day at Zagreb.

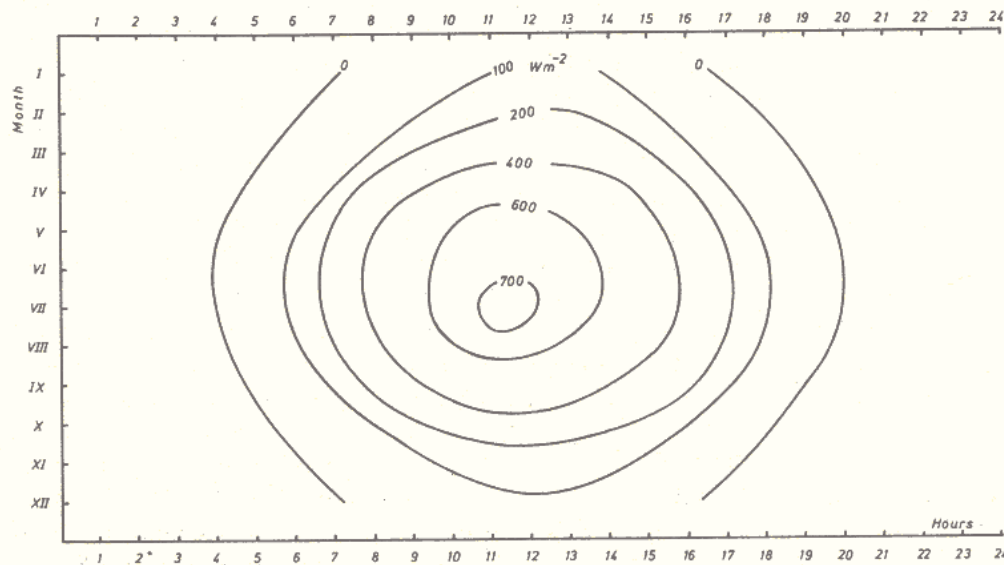


Figure 2. Isopleths of energy thresholds exceeded by global solar irradiation in 50 percent cases, i. e. lasting up to 30 minutes in a single hour of an average day at Zagreb.

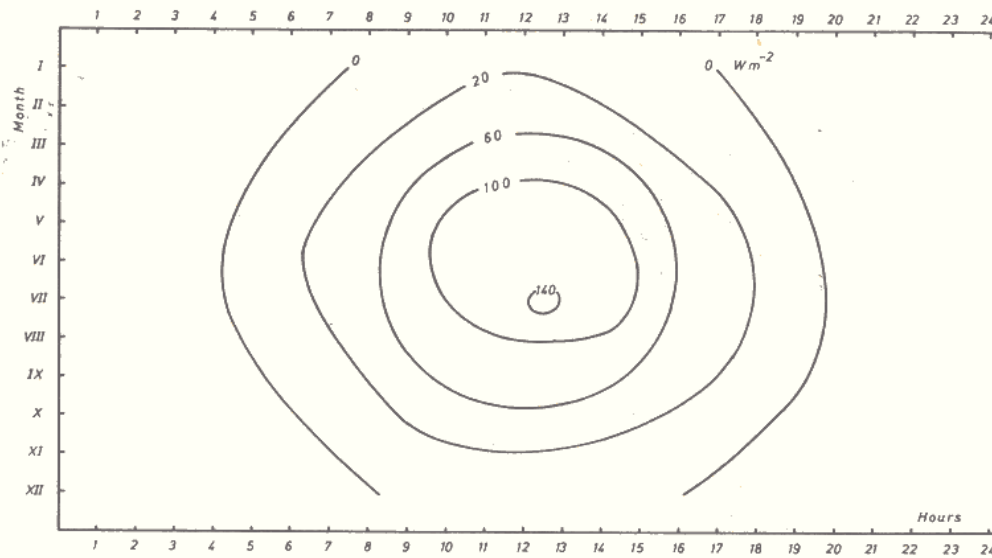


Figure 3. Isopleths of energy thresholds exceeded by global solar irradiation in 90 percent cases, i. e. lasting over 54 minutes in a single hour of an average day at Zagreb.

#### 4. Application of results

Data from Tab. 1 and the known methods (Jordan and Liu 1977, Kreider and Kreith 1981, Penzar 1985, Robinson 1966, Žic 1980) provided a basis for computing the amounts of solar energy received by planes inclined in the angles of 30, 40, 50, 60 and 90°. The energy amounts were used as inputs for the mathematical model simulating the work of a solar water heating system with the flat plate collectors in order to test the real gain of heating system. In the computer simulation, technical data for the collectors made by Rade Končar factory in Zagreb (1985) were used.

The months January, April and July are chosen for the illustration of the results in Figures 4 to 6. The high of the hatched part of the drawing shows the heat obtained if the simulation is done by global irradiation mean values, and the total high shows the heat if the simulation is done by energy levels from Tab. 1. In winter months (Fig. 4.) simulation by energy levels yields measurable heat quantity and at water temperature up to 50°C while simulation by mean values yields small quantities of heat with water temperature slightly over 20°C. The optimal tilt of the collector is about 60° then. The difference between the heat calculated in the two ways is considerably smaller in spring and autumn (Fig. 5). The optimal angle of tilt as this time is 30° to 40°. In the summer months (Fig. 6) the two ways of calculation of heat yield practically equal results if considered temperature level does not exceed 50°C. Most heat is then produced by collectors tilted at small angles.

The winter results obtained by the energy levels method were proved on an experimental installation in Zagreb. For this reason, the above shown special data proces-



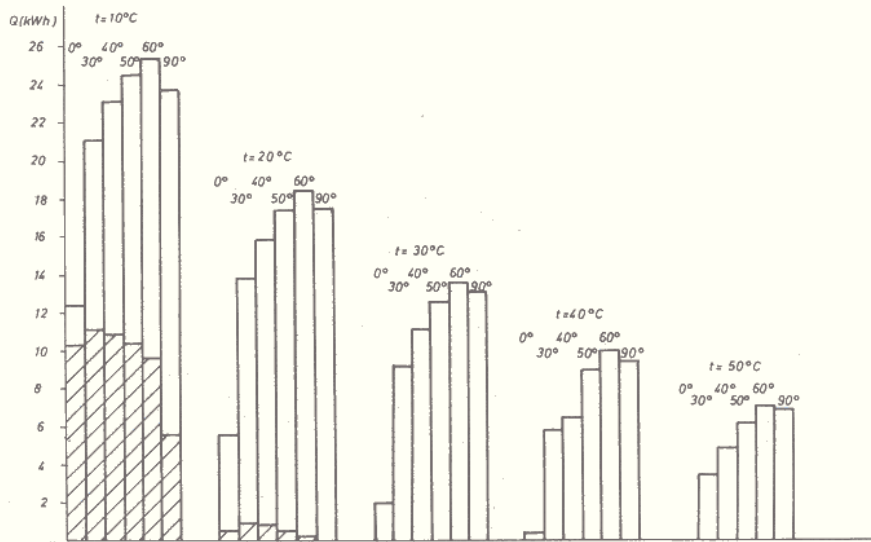


Figure 4. Energy  $Q$  available for heating of water to  $10^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$ , ...  $50^{\circ}\text{C}$  at Zagreb in January. The heat is produced by means of solar collector of  $1\text{ m}^2$  area, tilted southwards at different angles from  $0^{\circ}$  to  $90^{\circ}$ , ▨ results respecting energy levels, ▩ results from mean values.

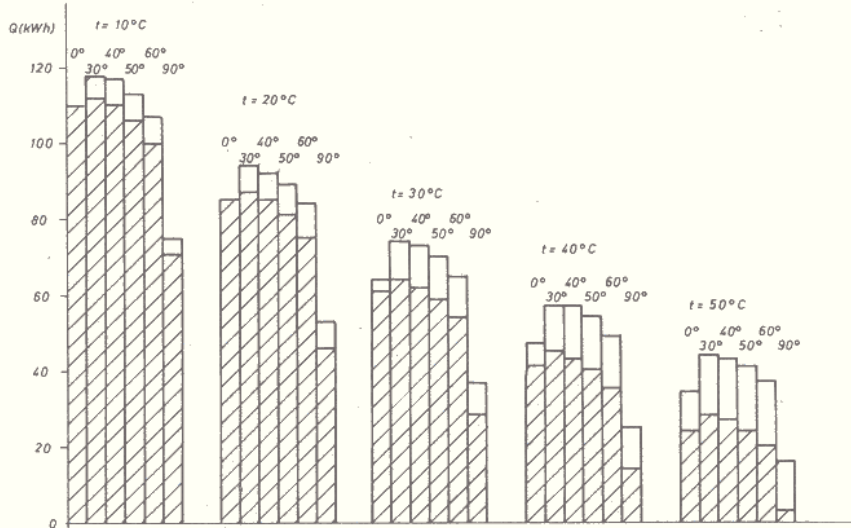


Figure 5. Energy  $Q$  available for heating of water to  $10^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$ , ...  $50^{\circ}\text{C}$  at Zagreb in April. The heat is produced by means of solar collector of  $1\text{ m}^2$  area tilted southwards at different angles from  $0^{\circ}$  to  $90^{\circ}$ , ▨ results respecting energy levels, ▩ results from mean values.

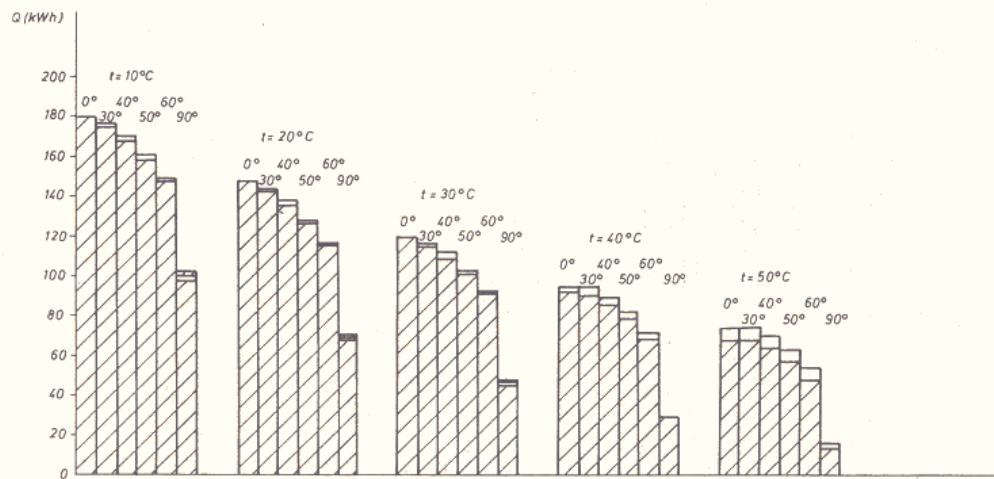


Figure 6. Energy  $Q$  available for heating of water to  $10^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$ , . . .  $50^{\circ}\text{C}$  at Zagreb in July. The heat is produced by means of solar collector of  $1\text{ m}^2$  area tilted southwards at different angles from  $0^{\circ}$  to  $90^{\circ}$ ,  $\square$  results respecting energy levels,  $\square$  results from mean values.

sing technique for solar energy application may be recommended instead of the technique used in regular meteorological practice. Besides, the results suggest that some part of the energy necessary to heat space in Zagreb in winter might be obtained from the solar irradiation.

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Authors' addresses: I. Penzar, Geophysical Institute, Faculty of Science, University of Zagreb, P. O. Box 224, 41001 Zagreb, Yugoslavia

M. Žic, Electrotechnical Institute, Rade Končar – Electrical Industries Products, 41001 Zagreb, Fallerovo šetalište 22, Yugoslavia