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The magnitude-intensity-focal depth relation for the earthquakes in the wider Dinara region

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The regression coefficients in the M-I_0-log h relation were estimated on the basis of local magnitudes (M), epicentral intensities (I_0) and focal depths (h) of 50 earthquakes which occurred in the wider region of Dinara mountain, Yugoslavia. The relation is

$$-M/3.18 + (\log h)/1.72 + I_0/3.63 - 1 = 0$$

Since the three-axes regression was utilized the resulting expression is valid for estimation of any of the variates on the basis of the observations of other two.

Relacija između magnitude, intenziteta i dubine žarišta za potrese u širem području Dinare

Određeni su koeficijenti u relaciji koja povezuje opažene magnitude (M), intenzitete u epicentru (I_0) i dubine žarišta (h) za 50 potresa u širem prostoru planine Dinare. Relacija glasi

$$-M/3.18 + (\log h)/1.72 + I_0/3.63 - 1 = 0$$

Kako je korišten postupak regresije koji sve tri varijable tretira simetrično, taj se izraz može upotrebljavati za procjenu bilo koje od triju veličina M, log h, I_0 na osnovi opažanja ostalih dviju.

1. Introduction

The empirical relations between earthquake magnitude, its intensity and depth of focus are used, for instance, by those studying the seismicity of some region to estimate the magnitude of large historic earthquakes from existing intensity maps. The engineering seismologist will often need it in order to predict the intensity of an earthquake on the basis of the maximum possible magnitude for a certain return period. Much more rarely (but see e.g. Davidson and Bodé (1987) and Reinbold and Johnston (1986)) such expressions are used to estimate the focal depth in cases when it is poorly constrained by the instrumental data. From the examples above, it may be seen that there is need for expressions which would consistently estimate intensity from earthquake magnitude, and vice versa. The relation which is in Yugoslavia almost exclusively used is the one proposed by Kárník (1969) for southern Italy, Balkans and Turkey:

$$M = (2/3)\, I_o + 1.7 \log h - 1.7 \tag{1}$$

Here I_o denotes the epicentral intensity in degrees of the MCS scale, h is the hypocentral depth in km and M is the local earthquake magnitude. However, it was observed that intensities are usually overestimated by using this formula for earthquakes in the littoral part of Croatia. Herak et al. (1988) have therefore proposed two relations derived on the basis of data collected in the catalogue of earthquakes for the wider Dinara region (1979-1987):

$$I_o = 1.06\, M - 1.52 \log h + 3.21 \tag{2}$$

$$M = 0.70\, I_o + 1.20 \log h - 1.42 \tag{3}$$

The coefficients in above formulas were obtained by regressions of I_o on (M, $\log h$) and of M on (I_o, $\log h$). As stated in Herak et al. (1988) it is very impractical to have to use different relations for I_o and M estimation. In this paper an effort will be made to derive the M–$\log h$–I_o relation by treating all three variates symmetrically.

2. The method

The data usually consist of N triplets ($M_i, h_i, I_{oi}, i = 1..N$). The observational errors for each of the observables are assumed to be normally distributed with the zero mean and the standard deviation ($\sigma_{M_i}, \sigma_{h_i}, \sigma_{I_i}$). Since uncertainties are present in all three coordinates, the three-axes regression is one of the ways to properly set a problem of finding the best values (in the least squares sense) for the coefficients in the M–$\log h$–I_o formula. As all three variates are to be treated symmetrically, let's write the equation of the regression plane in the segment form:

$$M/A + (\log h)/B + I_o/C = 1 \tag{4a}$$

or

$$M/A + z/B + I_o/C = 1 \tag{4b}$$

with $z_i = \log h_i$. Partly following Tarantola (1987), we shall aim to minimize

$$G = \sum_{i=1}^{N} (M_i / A + z_i / B + I_{oi} / C - 1)^2 \,/\, \sigma_i^2 = \Sigma\, (\delta_i^2 / \sigma_i^2) \tag{5}$$

where σ_i^2 is the variance of the i-th triplet. Using the well known approximation for uncorrelated variables

$$\sigma^2\,[F(f_1, f_2, f_3)] = \left(\frac{\partial F}{\partial f_1}\sigma_{f_1}\right)^2 + \left(\frac{\partial F}{\partial f_2}\sigma_{f_2}\right)^2 + \left(\frac{\partial F}{\partial f_3}\sigma_{f_3}\right)^2 \tag{6}$$

σ_i^2 is defined as:

$$\sigma_i^2 = \left(\frac{\partial\delta}{\partial M}\sigma_{M_i}\right)^2 + \left(\frac{\partial\delta}{\partial z}\sigma_{z_i}\right)^2 + \left(\frac{\partial\delta}{\partial I_0}\sigma_{I_i}\right)^2$$

$$\sigma_i^2 = \left(\frac{\sigma_{M_i}}{A}\right)^2 + \left(\frac{\sigma_{z_i}}{B}\right)^2 + \left(\frac{\sigma_{I_i}}{C}\right)^2 \qquad (7)$$

σ_z appearing in (7) is the standard deviation of the variable $z = \log h$. It is, however generally not correct to use (6) and conclude $\sigma_z = \sigma_h (\log e)/h$. This is due to the fact that h_i may take only positive values. It is reasonable even not to allow depths less than 1 km, because of the very rapid and unrealistic decay of $\log h$ for $0 \leq h \leq 1$. The consequence of this truncation is considerable distortion of the distribution of $z = \log h$ for small h and large σ_h, which is difficult to express analytically. The empirical table for the standard deviation (σ_z) of the distribution of $z = \log h$ (presented in the Appendix, table A2) when h is normally distributed was therefore used to estimate σ_z when h and σ_h were given.

There are numerous ways to find the values of A, B and C which minimize G in (5) for a particular data set (see for example Tarantola, 1987). The most direct, and a very informative one, is the simple grid search method. It consists in computing the G value for a number of fixed points in (A, B, C) space, and adopting as the solution those values A_0, B_0 and C_0 where G attains the minimum. In our case, there are three unknown coefficients, but the grid search may nevertheless be performed in only two dimensions since one of the unknowns (say C) is always expressable as a function of the other two by using the condition on the expected value of δ:

$$\mathrm{E}(\delta) = 0 \qquad (8)$$

It follows

$$\mathrm{E}(M)/A + \mathrm{E}(z)/B + \mathrm{E}(I_0)/C - 1 = 0$$

$\mathrm{E}(M)$ and $\mathrm{E}(I_0)$ are simply the mean observed values of M and I_0. $\mathrm{E}(z)$ is a function of the mean of the afore mentioned distribution of the variable z, i.e.

$$\mathrm{E}(z) = \frac{1}{N} \sum [z_i + u(h_i, \sigma_{h_i})] \qquad (9)$$

The values $u_i(h, \sigma_h)$ are tabulated in table A1 in the Appendix. They are defined as differences between the true mean of the truncated distribution of $\log h$ (when h is normally distributed with mean h_i and standard deviation σ_h) and z_i. From (8) and (9) it follows that C is given for any pair (A, B) by

$$C = \frac{\Sigma I_0}{N - (\Sigma M)/A - [\Sigma(z+u)]/B} \qquad (10)$$

The advantage of using a grid-search method for locating the minimum of the misfit function G is that the G-value is then known at a number of equally spaced points defining the grid. Using the approach described in Sambridge and Kennett (1986) (see also Buland, 1976; Herak, 1989) it is then possible to define the $p\%$ confidence region contour lines which enclose the values of regression coefficients regarded by data as jointly reasonable at the specified level.

3. The data and results

The procedure outlined in section 2. was applied to derive the M–log h–I_o relation for the wider region of Dinara mountain, Yugoslavia. The data taken into account come from earthquakes which occurred in the epicentral areas of Knin–Bosansko Grahovo, Svilaja mt, Dinara mt, Ravni Kotari and karst fields of Imotski, Sinj, Livno, Glamoč and Kupres. The data set consists of reliably estimated local magnitudes (M), maximum observed intensities (I_{max}), focal depths (h) and horizontal distances (d) from the place where the intensity was observed to the epicenter (see Table 1). For all data the restriction $d \leq 25$ km was adopted. The most of data (for earthquakes between 1979 and 1987) were already used by Herak et al. (1988) to obtain expressions (2) and (3). Adding also the available data for events in the period 1955–1978 a total of $N = 50$ triplets (M, I_o, h) was obtained. Just as in Herak et al. (1988) the epicentral intensity I_o was estimated from I_{max}, d and h by using the expression (Sponheuer, 1960)

$$I_o = I_{max} + 3 \log (R/h) + 3 \log e\, \alpha(R - h) \tag{11}$$

$$R^2 = d^2 + h^2$$

$$\alpha = 0.002 \text{ km}^{-1}$$

The standard deviations σ_h for the focal depth and σ_d for the horizontal distance were taken from the available catalogues and ranged between 2.5 and 10 km. σ_M and $\sigma_{I_{max}}$ were set to be equal for all events – 0.3 magnitude units and 0.5°MCS respectively. Using expressions (11) and (6) σ_{I_o} was obtained for each event.

The grid search then yielded $A_o = -3.18$, $B_o = 1.72$ with $\Sigma M = 187.15$, $\Sigma I_o = 299.83$, $\Sigma(z + u) = 45.00$. C_o is then given by (10), and equals $C_o = 3.63$, so that the M–log h–I_o relation is

$$-M / 3.18 + (\log h) / 1.72 + I_o / 3.63 - 1 = 0 \tag{12}$$

This relation is presented for three different depths, together with the data on Figure 1. Rewriting (12) as

$$M = 0.88\, I_o + 1.85 \log h - 3.18 \tag{13}$$

$$I_o = 1.14\, M - 2.11 \log h + 3.63 \tag{14}$$

$$\log h = 0.54\, M - 0.47\, I_o + 1.72 \tag{15}$$

we obtain relations to be used to estimate one of the parameters on the basis of other two.

Table 1. The basic parameters of 50 earthquakes used to derive regressions (13) — (15). All hypocenters were located microseismically (Herak et al. (1988), Herak D. (unpublished)), except the one of 15.09.1985. for which the macroseismic epicentre was used. The local magnitudes (M) were obtained from the records of station ZAG. Intensities I_{max} were taken from the archive data of the Geophysical Institute, Zagreb.

Date	Epicentre °N	Epicentre °E	Depth km	M	I_{max} °MCS	d km
15.09.1955.	43.7	16.6	6.	3.8	6.0	0
05.08.1970.	43.941	16.021	16.5	4.2	6.0	5
07.09.1970.	43.991	16.108	11.1	5.3	8.0	0
01.04.1972.	43.947	16.086	16.8	3.9	6.0	0
23.05.1974.	43.426	17.186	6.6	4.1	6.5	4
22.02.1976.	44.199	15.891	12.2	4.2	6.0	0
13.01.1977.	43.563	17.249	8.9	4.8	6.0	15
30.01.1977.	44.020	16.033	1.0	3.7	5.0	8
31.01.1977.	44.092	16.056	7.2	3.4	5.0	10
09.02.1978.	44.161	16.967	7.9	4.5	6.0	14
17.12.1978.	43.416	17.385	16.3	4.6	6.5	0
08.01.1979.	44.209	15.898	6.2	4.0	5.5	5
04.07.1979.	44.031	16.676	11.3	4.8	6.5	18
25.07.1979.	43.503	17.329	16.7	4.2	5.0	10
21.11.1979.	44.229	15.898	10.5	3.8	5.0	10
02.01.1980.	43.940	16.284	7.8	3.1	3.5	20
29.08.1980.	43.999	17.000	3.1	4.0	5.0	15
28.12.1982.	43.974	16.167	3.9	2.6	4.0	10
19.06.1983.	43.877	16.144	3.6	3.5	5.5	8
13.08.1983.	44.288	16.051	1.4	3.2	4.0	18
28.12.1983.	43.971	15.949	2.9	2.9	4.0	20
18.03.1984.	44.512	17.189	1.0	4.2	6.0	5
22.04.1984.	43.861	15.442	8.0	3.1	5.0	7
03.05.1984.	43.927	16.513	4.2	2.9	4.0	15
04.05.1984.	44.033	15.940	16.0	3.1	5.0	15
06.09.1984.	43.942	17.257	8.3	4.6	7.0	0
11.11.1984.	43.518	16.243	1.4	2.6	4.5	5
21.04.1985.	43.837	16.527	18.8	4.3	5.0	3
28.09.1985.	43.872	16.588	14.5	3.9	6.0	0
16.01.1986.	43.988	16.132	6.1	3.0	4.5	0
29.04.1986.	43.649	16.560	17.8	4.1	6.0	9
29.04.1986.	43.719	16.631	14.4	3.0	4.0	9
06.11.1986.	43.988	16.337	3.1	3.2	4.0	11
25.11.1986.	44.068	16.317	13.2	5.5	7.5	0
26.11.1986.	44.092	16.603	9.3	3.4	4.5	22
26.11.1986.	44.109	16.373	14.1	2.7	4.0	9
26.11.1986.	44.058	16.269	7.3	3.3	4.5	5
26.11.1986.	44.052	16.219	4.7	3.1	5.0	5
27.11.1986.	44.057	16.321	14.5	4.4	6.0	10
27.11.1986.	44.072	16.288	6.1	3.6	5.0	5
28.11.1986.	44.104	15.343	9.0	3.9	5.5	14
06.12.1986.	44.111	15.451	14.6	3.0	5.0	3
08.12.1986.	43.896	16.360	22.2	2.9	4.0	7
24.12.1986.	43.997	16.220	9.7	4.7	6.5	0
08.01.1987.	43.687	16.914	1.1	4.2	6.0	5
21.01.1987.	43.480	17.084	8.5	3.1	4.5	8
15.02.1987.	44.052	16.279	12.3	2.6	4.0	10
15.02.1987.	43.887	16.838	10.1	3.7	4.5	25
24.03.1987.	44.082	16.424	8.4	4.4	6.0	8
08.11.1987.	44.028	16.324	8.8	4.1	5.5	7

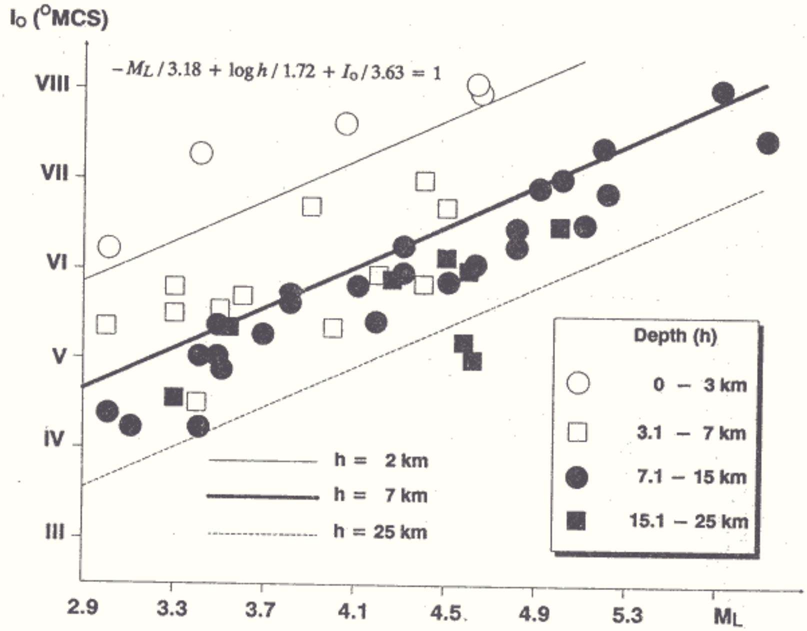

Figure 1. The data used to estimate expression (12). The straight lines are computed by the relation (14) for the focal depths of 2, 7 and 25 km.

The confidence regions for A, B and C are presented on Figure 2. It may be seen that pronounced correlation exists for coefficients A and C. Also, one may note that the contour lines are of the form of deformed ellipses which is the consequence of the non-normal distribution of $z = \log h$. From the plots of Figure 2 it is possible to estimate the standard deviations σ_A, σ_B and σ_C, as

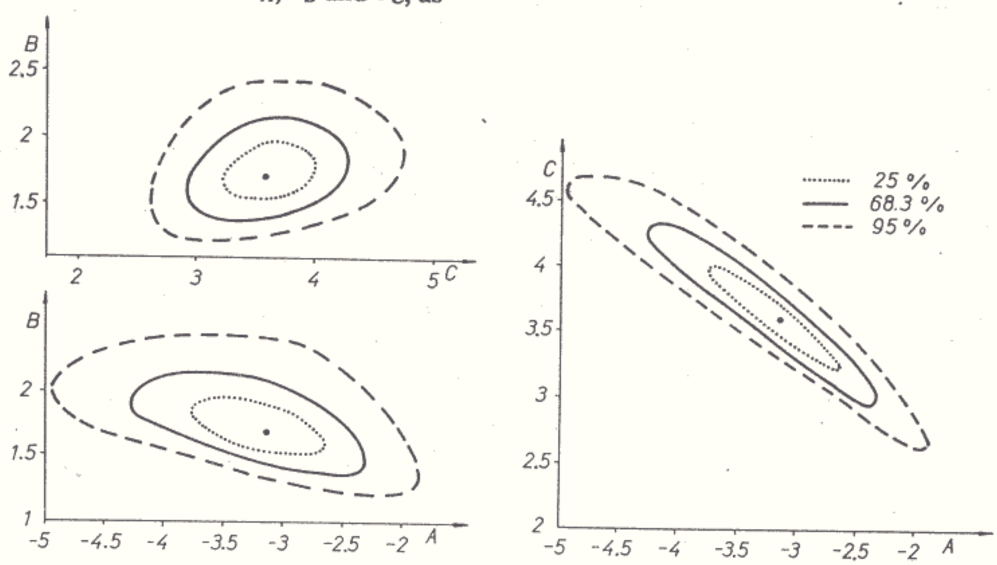

Figure 2. The 25%, 68.3% and 95% confidence regions for the coefficients A_o, B_o and C_o in (12).

$$\sigma_A = 1.00,\ \sigma_B = 0.39,\ \sigma_C = 0.70.$$

The standard errors for M, I_0 and z, estimated on the basis of (13) – (15) are

$$s_M = 0.36,\ s_{I_0} = 0.42\ ^{\circ}\mathrm{MCS},\ s_z = 0.20,$$

and the coefficients of correlation are

$$r_M = 0.87,\ r_{I_0} = 0.92,\ r_z = 0.82\,.$$

From these, the coefficients of determination are given as

$$D_M = 76\%,\ \ D_{I_0} = 84\%,\ D_z = 68\%.$$

This means that the regresion equation (13) explains 76% of the observed variation in magnitudes on the basis of intensity and focal depth data. The same applies to intensity (with 84% for formula (14)) and to $z = \log h$ (with 68% explained variation, expression (15)).

Testing the regresion equations (13) – (15) with F-test, we find them significant at a level of more then 99.5%.

4. Conclusions

The result of this study, the relation (12), was derived on the basis of three-axial regression which treats M, I_0 and z in a symmetrical way. This fact enables using the same expression for estimation of one of the quantities on the basis of other two. It seems meaningful to use the same procedure whenever one has to deal with more than one observables which are known with some error. The examples in the field of engineering seismology are numerous, and include all empirical attenuation functions for displacement, velocity, acceleration etc. These relations are usually derived by using simple one dimensional regression methods (usually least squares), without taking the uncertainties in observed values into account. Since the logarithmic dependences are often assumed it is hoped that the tables presented in Appendix will be found useful also in such applications.

It should be pointed out that the equation of the macroseismic field (11) was used to reduce the observed intensities to the epicentral intensity in its original form as proposed by Sponheuer (1960). The choice of some other intensity attenuation law may have considerable influence on the resulting regression coefficients. Also, in this paper the source size was not taken into account. The use of the distance from the causative fault instead of d in (11) would probably reduce the scatter of data and improve the reliability of estimated parameters.

References

Buland, R. (1976): The mechanics of locating earthquakes, Bulletin of the Seismological Society of America, **66**, 173-187.

Davidson, F.C. Jr. and Bodé, M.J. (1987): A note on the December 1986 - January 1987 Richmond, Virginia, felt earthquake sequence, Seismological Research Letters, **58**, 73-80

Herak, D., Herak, M. and Cabor, S. (1988): Some characteristics of the seismicity and the earthquake catalogue of the wider Dinara mountain area (Yugoslavia) for the period 1979-1987, Acta Seismologica Iugoslavica, **14**, 27-59 (in Croatian with English abstract).

Herak, M. (1989): HYPOSEARCH – An earthquake location program, accepted for publication in Computers & Geosciences. (in print)

Kárník, V. (1969): Seismicity of the European area, Part 1, D. Reidel Publishing Company, Dordrecht, p. 68.

Reinbold, D.J. and Johnston, A.C. (1986): Historical seismicity in the Southern Appalachian seismic zone, USGS Final Technical Report, Contract No. 14-08-0001-21902, 40 pp.

Sambridge, M.S. and Kennett, B.N.L. (1986): A novel method of hypocentre location, Geophysical Journal of the Royal Astronomical Society, **87**, 679-697.

Sponheuer, W. (1960): Methoden zur Herdtiefen Bestimung in der Makroseismik, Freiburg Forschunghefte, **C 88**.

Tarantola, A. (1987): Inverse problem theory, Elsevier, Amsterdam, pp. 613.

Appendix:

The values of $u(\bar{h}, \sigma_h)$ and $\sigma_z(\bar{h}, \sigma_h)$ (see section 3) are tabulated in tables A1 and A2 respectively. It was assumed that the distribution of h is modified truncated normal with mean $\bar{h}$ and standard deviation σ_h. The modification consists in truncating the h values at $h = 1$ by assigning $h_i = 1$ for each $h_i < 1$. For each pair $(\bar{h}, \sigma_h)$ 30000 normally distributed values were generated, truncated, and $z_i = \log h_i$ ($i = 1, \ldots, 30000$) were calculated. The mean m_z and the standard deviation σ_z were then estimated from z_i values for each $(\bar{h}, \sigma_h)$.

Table A1. The values of $u(h, \sigma_h) = m_z - \log h$ for selected values of h and σ_h.

$\bar{h}$ (km)	$\sigma_h = 2$	4	6	8	10
1	-0.182	-0.274	-0.338	-0.386	-0.421
2	-0.000	-0.051	-0.094	-0.131	-0.168
3	0.051	0.040	0.019	0.004	0.034
4	0.052	0.085	0.079	0.059	0.035
5	0.040	0.096	0.106	0.105	0.093
6	0.028	0.095	0.116	0.130	0.134
7	0.020	0.081	0.119	0.141	0.154
8	0.017	0.069	0.117	0.143	0.158
9	0.012	0.058	0.106	0.141	0.159
10	0.009	0.045	0.096	0.139	0.161
11	0.008	0.035	0.087	0.122	0.159
12	0.007	0.032	0.073	0.117	0.158
13	0.006	0.025	0.063	0.109	0.147
14	0.005	0.021	0.058	0.099	0.138
15	0.004	0.018	0.048	0.085	0.128
16	0.003	0.015	0.040	0.078	0.115
17	0.003	0.014	0.033	0.069	0.108
18	0.003	0.011	0.031	0.062	0.097
19	0.003	0.009	0.024	0.056	0.090
20	0.002	0.009	0.024	0.049	0.083
21	0.002	0.007	0.020	0.041	0.074
22	0.002	0.008	0.018	0.038	0.065
23	0.002	0.007	0.019	0.035	0.060
24	0.002	0.006	0.016	0.030	0.056
25	0.001	0.006	0.014	0.027	0.049
26	0.001	0.005	0.013	0.025	0.046
27	0.001	0.005	0.011	0.022	0.041
28	0.001	0.004	0.011	0.020	0.037
29	0.001	0.004	0.011	0.018	0.033
30	0.001	0.004	0.009	0.017	0.032
31	0.001	0.003	0.009	0.017	0.031

Table A2. The values of σ_z for selected values of h and σ_h.

$\bar{h}$ (km)	$\sigma_h = 2$	4	6	8	10
1	0.230	0.330	0.396	0.445	0.482
2	0.264	0.352	0.414	0.458	0.495
3	0.268	0.366	0.422	0.467	0.502
4	0.245	0.365	0.430	0.475	0.510
5	0.206	0.355	0.428	0.475	0.517
6	0.170	0.338	0.421	0.476	0.517
7	0.141	0.313	0.410	0.471	0.512
8	0.121	0.285	0.394	0.460	0.506
9	0.104	0.256	0.373	0.447	0.502
10	0.092	0.226	0.353	0.436	0.488
11	0.082	0.199	0.330	0.417	0.480
12	0.075	0.179	0.302	0.399	0.470
13	0.068	0.157	0.279	0.381	0.455
14	0.064	0.143	0.258	0.362	0.439
15	0.060	0.130	0.233	0.336	0.424
16	0.056	0.119	0.213	0.314	0.403
17	0.052	0.110	0.192	0.297	0.385
18	0.049	0.104	0.180	0.277	0.365
19	0.046	0.096	0.163	0.258	0.347
20	0.044	0.091	0.154	0.237	0.332
21	0.041	0.086	0.142	0.220	0.308
22	0.040	0.082	0.134	0.207	0.291
23	0.038	0.079	0.128	0.194	0.273
24	0.037	0.075	0.120	0.179	0.261
25	0.035	0.072	0.115	0.168	0.245
26	0.034	0.069	0.108	0.159	0.229
27	0.032	0.066	0.104	0.148	0.214
28	0.031	0.064	0.100	0.142	0.200
29	0.030	0.061	0.095	0.135	0.192
30	0.029	0.059	0.091	0.131	0.185
31	0.028	0.057	0.087	0.125	0.178

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