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Fuzzy modelling in air protection

Damir Rumenjak[1] and Siniša Štambuk[2]

[1] Ministry of Environmental Protection, Physical Planning and Construction, Zagreb, Croatia

[2] State Inspectorate, Zagreb, Croatia

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It is presented how classical modelling in environmental protection, particularly air quality modelling from area pollution sources, could be replaced by fuzzy modelling. The results of such modelling are fuzzy sets that could be used in hybrid models for approximate reasoning and decision making support. The futures work on environmental standards representation and construction the linguistic variables for the purpose is also explained.

Keywords: Fuzzy sets, membership function, linguistic variables, air quality modelling, eulerian box model, environmental impact assessment, environmental standards

1. Introduction

Fuzzy sets (Zadeh, 1965) are useful tool for describing uncertainty. The difference between fuzzy and classical or crisp sets is that for the former, membership function is in interval [0,1], instead of being 1 or 0. The fuzziness, as a set property, is described as a matter of human perception and reasoning that is impossible to cross (Dimitrov et al., 2002). The construction of kind of fuzzy sets that describes property of interest – a linguistic variables of acceptability for environment, could utilize the decision making (or deciding between the variants of interventions in environment) close to right based paradigm of decision-making, primarily for using in environmental impact assessment (EIA). In Figure 1 the general linear form of linguistic variable, using indicators of environmental impact as arguments of membership function, $\mu(i)$, is shown by two partitions.

In most cases fuzzy sets are exploited for approximate reasoning in fuzzy logic systems, avoiding mathematical modelling (Cox, 1999). But the development of the fuzzy computational approach, in the wider context of soft computing idea, leads to possibilities of fuzzifying variables and equations of various models. The methods are proposed for solving fuzzy equations exploiting the extension principle, including the most advanced methods like neural

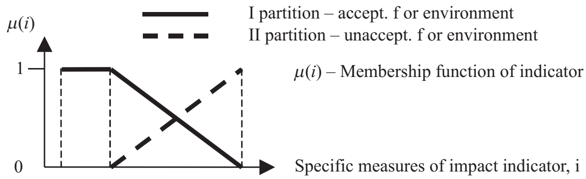

Figure 1. Linguistic variable: acceptability for environment with two partitions.

networks (Buckley et al., 2002). The results could be also used in hybrid or knowledge-based and modelling systems (Demico et al., 2004), that unify both approaches.

There are few ways discussed of incorporating results of mathematical modelling mentioned in knowledge-based or expert systems for environmental protection. One is to include results of fuzzy modelling, which could be either fuzzy or defuzzified, in system for approximate reasoning based on fuzzy sets (Rumenjak et al., 2005). The second way could be construction of the appropriate linguistic variables, which is of the interest for the future work.

2. Modelling of air quality with box models in EIA – the history

Since 2000, when environmental legislation asks the use of models in EIA procedures in Croatia, there were several environmental impact assessment studies using Eulerian box or compartment models for predicting the air quality but only for the extraction of mineral resources (non-metallic mineral raw materials, usually building stone and gravel). The reason why this type of model is preferred by the authors of studies over other types of model for area source pollution, Lagrangian or modified Gaussian, could be explained, except

Table 1. Emissions into air from area sources for most important pollutants in quarrying obtained by American EPA models (including internal transport and stone dressing

Location – region or city in Croatia	Q_m (g/h)			Effective area of pit (ha)	Eff. width (m)	Annual production of stone aggregates (t)
	PM–30*	SO_2	NO_x			
Rakalj–Istra	101 398	4207.8	40 101.1	16	400	351 000
Vrlika, Dalmatia	66 300	4106	17 008.6	16	400	270 160
Debeljača, Split	25 130	1 330.4	9 827.4	23	480	162 000
Litićev vrh, Lika	355 841	18 838.5	139 155	66	800	2 700 000
Klis kosa, Split	53 110	1432	10 982	45	500	200 000

*according to some assumptions in EIA studies, the part of PM–10 particles is about 30% per total mass of PM–30 particles, including primary particulates from gas phase by the simplicity of the model, also by the demands of stakeholders and regulators involved in assessment that evaluation of air quality has to be done on the borders of exploitation fields, independently of the fact whether is any sensitive target in the vicinity of the pits. Anyway, it gives also one example of using Lagrangian type or column type of box model (EIA Držimurec, 2003), because of small working area (compared with total area occupied) and possible impacts on some longer distance from the borders of exploitation pit.

Some values for emissions (Q_m) in quarrying, entering the Eulerian box model, is shown in Table 1. The values are taken from the environmental impact assessment studies for particular mining project.

Emission values were calculated by United States environmental protection agency (US EPA) emission models, using emission factors. Those models account for emissions of technological operations in mining.

In almost all cases, one box approach (Jacobi, 1999) was used in modelling. But there is also example when two box model approaches is used for modelling because of significant difference in terrain levels in the pit (EIA Rakalj, 2001).

3. The characteristics of Eulerian type box model used in modeling

The box model, used in environmental impact assessment studies for extraction of mineral resources, has the following differential description for the change of concentration of pollution substance with no reaction, degradation or sedimentation loss of substances assumed:

$$\frac{dC_i}{dt} = \frac{q_i}{H_m} + \frac{U}{\Delta x}(C_{i0} - C_i) \tag{1}$$

The solution is found for steady state condition, putting $\frac{dC_i}{dt} = 0$, $C_i = C_{ss}$ and $C_{i0} = C_0$. When one box approach is used, it is assumed that Δx is equal to the length of the box (L) in the wind direction. Introducing the length of the box into solution of equation (1) utilizes the use of total emission of pollution substance from area sources Q. According to assumptions, the following formula for the steady state concentration is then used:

$$C_{ss} = C_0 + \frac{Q_m}{UWH_m} \tag{2}$$

Mostly, for the practical purposes, the term C_0 is neglected and the formula used in modelling is:

$$C_{ss} = \frac{Q_m}{UWH_m} \tag{3}$$

This also enables skipping the right determination of the area, except the width W, which has to be determinate as effective width.

For box models, the most intricate variables are wind velocity and mixing height (which also explains why the fuzzy sets are convenient for expressing them). The direction of wind velocity is assumed to be of no significance, because of the general demand of stakeholders in EIA that concentration values of pollutants on the whole border of the pit have to be beyond the ambient limits prescribed by legislation.

In EIA for quarrying, the wind velocity is taken from wind diagrams (wind roses), often as average or minimal value recorded. Only velocities on 3 to 10 m above the ground were used. The increasing of velocities with height, which is usually calculated according to the formula for the height below 200 m:

$$U(z) = U_{10}(\frac{z}{10})^{p} \tag{4}$$

is not considered, as well as wind velocities at lower levels according to von Karman equation (Heinsohn et al., 1999). The justification for this attitude is commonly found in formula used for the steady state concentration, which shows general increase of U and resulting decrease of C_{ss} with height.

The mixing heights could be determined using the diagrams of annual air temperatures profile combining with dry lapse rate. Another technique, based on thermal flux from the ground and using the temperature on the surface (in practice measured 2 cm beneath the cover), is recommended, with resulting simpler temperature profile (Reible, 2000). It is difficult to obtain the mixing heights for each location of pits, and often regional approach is used.

For deciding the sources of emission (technological operations) to include them in inventory of emissions sources for box model, the criteria of residence time is often used as $t > 3\tau_r$, where t is duration of continuous or semi continuous operation, τ_r is residence time calculated by $\tau_r = L/U$. according to criterion $t > 3\tau_r$, only the blasting operation in mining is often omitted, because of impossibility of achieving conditions necessary for the steady state box modelling.

4. Fuzzy relations for air quality modelling

For the purpose of fuzzy modelling, the arguments for membership functions have to be chosen and relation between fuzzy and crisp variables in modelling established. As crisp variable total emission value Q_m is chosen, because of its clear control role later on in decision-making. The problem, often associated with the use of parameter Q_m, is that existing emission standards prescribe only emission values from point sources and not values from area sources. At present, only concentration limits (or ambient values) from legislation are on disposal for some evaluation of impacts of emissions from area sources. So, there is no other source of data for area pollution emissions that could be used, except modelling.

Variables of equation (3), U and H_m, are left for fuzzification. The parameter W, that is also possible to fuzzify, is handled here as crisp constant for the reason of simplicity. This approach could be taken as convention for various classes of area pollution sources (e.g. small, medium, high).

The fuzzy form of equation (3), according to usual notation used in fuzzy arithmetic, is:

$$\widetilde{C}_{ss} = \frac{Q_m}{\widetilde{U}\, W \widetilde{H}_m} \tag{5}$$

where, $\widetilde{C}_{ss}$, $\widetilde{U}$, $\widetilde{H}_m$ are fuzzy numbers.

5. The fuzzification of box model

A possibility theory, as analogy and alternative to probability theory, was developed by Zadeh, and it is also serving for expressing imprecise probabilities with fuzzy sets (Nguyen et al., 1997). The analogy of both theories is based on complementarity of the concepts of probability and possibility of events.

It is possible to use statistical data for construction of fuzzy sets which describe the possibility distributions. The logic behind such transforms is given (Verma, 2007). A simple transform to possibility distribution of Gaussian type is proposed (Ross, 1995):

$$\mu(x) = \exp\left[\frac{-(x - x_0)^2}{\sigma^2}\right] \tag{6}$$

where x is in domain of real numbers and $\mu(x)$ is in interval (0,1]. The similar transforms for other probability distributions are possible.

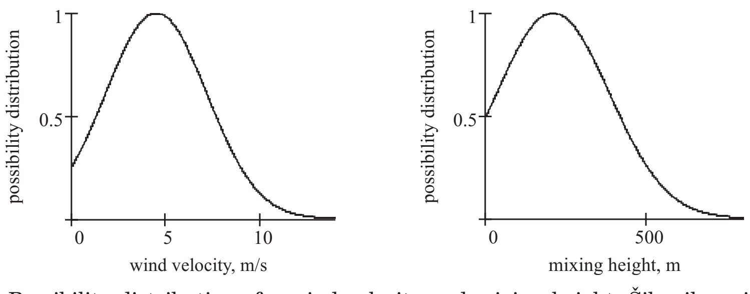

Figure 2. Possibility distributions for wind velocity and mixing height, Šibenik region (period 1951–1970).

The statistical data, chosen here for the construction of fuzzy sets, are wind velocities and mixing heights from the equation (3). The possibility distribution for mixing height calculated by the formula (6) is shown in Figure 2. For the construction, the annual distribution of surface temperatures, including the spectra of diurnal change, is used (Penzar, 2001). possibility distribution for wind velocities is shown (Figure 2), based on actual wind roses from the same reference.

The formula for $\widetilde{C}_{ss}$, using interval arithmetic and calculating the fuzzy interval for $\widetilde{C}_{ss}$, by fuzzy mathematical technique known as α–cut or $[C_{ss,1}(\alpha), C_{ss,2}(\alpha)]$, is:

$$[C_{ss,1}(\alpha),\, C_{ss,2}(\alpha)] =$$

$$= \frac{Q_m}{\left[U_0 - \left|(-\sigma^2{}_u \ln\alpha)^{\frac{1}{2}}\right|, U_0 + \left|(-\sigma^2{}_u \ln\alpha)^{\frac{1}{2}}\right|\right]}$$

$$\cdot \frac{1}{\left[H_{m0} - \left|(-\sigma^2{}_{H_m} \cdot \ln\alpha)^{\frac{1}{2}}\right|, H_{m0} + \left|(-\sigma^2{}_{H_m} \cdot \ln\alpha)^{\frac{1}{2}}\right|\right] \cdot W} \tag{7}$$

where U_0 and H_{m0} are wind and mixing height central values with membership of 1, respectively. The parameter *W*, which stays as a crisp number, has to be included in equation (7) according to the rules of interval arithmetic:

$$[C_{ss,1}(\alpha),\, C_{ss,2}(\alpha)] =$$

$$= \frac{Q_m}{\left[U_0 - \left|(-\sigma^2{}_u \ln\alpha)^{\frac{1}{2}}\right|, U_0 + \left|(-\sigma^2{}_u \ln\alpha)^{\frac{1}{2}}\right|\right]}$$

$$\frac{1}{\left[W \cdot H_{m0} - \left|(-\sigma^2{}_{H_m} \cdot \ln\alpha)^{\frac{1}{2}}\right|, W \cdot H_{m0} + \left|(-\sigma^2{}_{H_m} \cdot \ln\alpha)^{\frac{1}{2}}\right|\right]} \tag{8}$$

The equation (8) does not have classical solution beacuse, after applying rules of interval arithmetics, the solution is of type:

$$[C_{ss,1}(\alpha)\; C_{ss,2}(\alpha)] = \left[\frac{Q_m}{b(\alpha)d(\alpha)}, \frac{Q_m}{a(\alpha)c(\alpha)}\right] \tag{9}$$

where $a(\alpha)$, $b(\alpha)$, $c(\alpha)$ and $d(\alpha)$ are α–cuts at boundaries of intervals of fuzzy members of equation (5). The solution (9) is general interval α–cut approximate solution. The solution of equation for possibility distributions from

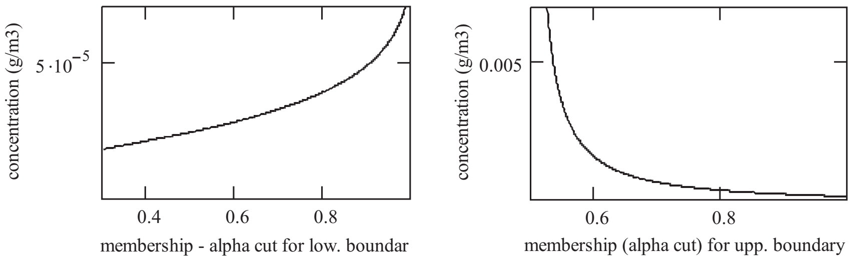

Figure 3. Possibility distributions for concentrations – solution of interval equation (8).

Figure 2, with conventional width W = 500 m and Q_m = 40 g s^{-1}, which is actual for NO_x emissions in modelled situations (Table 1), is shown in Figure 3.

Resulting distribution could be taken as stochastic and treated, applying suitable defuzzification techniques, for the imprecise probability expectations. But, for the purpose of decision making, it is better to use them as fuzzy numbers. After development of expert-systems with new demands on decision making in environmental protection, it is becoming difficult to use results from classical numerical modeling (both deterministic and stochastic). It is because expert systems use qualitative approach based on approximate reasoning and implication subsystems (Pilkey et al., 2007). Anyway, such systems became very sophisticated (Pastakia, 1998). The fuzzy sets based modelling, including construction of linguistic variables, is one of the approach to the solution of the problem by combining both types of models.

6. Method of setting linguistic variables based on fuzzy standard representation

Linguistic variables, having appropriate arguments and membership functions, are representing some property of interest for decision making. Calculation of membership values of I partition for linguistic variable: acceptability for environment (Figure 1), based on obtained possibility distributions of concentrations, is recommended by formula:

$$\mu(Q_m) = f\left(\frac{\int\limits_{CS} \mu(C_{ss})dC_{ss}}{\int\limits_{NCS} \mu(C_{ss})dC_{ss}}\right) \tag{10}$$

In above equation *CS* is denoting the fuzzy region of possibility distribution on compliance side of environmental standard, *NCS* denoting the fuzzy region on the non-compliance side of environmental standard, *f* becomes veristic function. The standard could be given in crisp (either *ad hoc* or statistical) or fuzzy representation.

At present no definition for *f* from equation (10) is developed. So, the brief description of the procedure which is under developing is given here. The function *f* obviously has to be achieved according to some form of consensus or expert choice, as it is usual for hybrid systems.

Linguistic variable (acceptability partition) is simply constructed with membership values $\mu(Q_m)$ and with Q_m as argument. Similarly, the II partition of LV: unacceptability for environment (Figure 1) could be constructed by changes of the formula (10) or simply in correlation with acceptability partition obtained before.

Setting of environmental standards is always matter for dispute (Barnett et al., 1997). In Table 2 and Table 3 the usual representations of air quality standard are given. It is important to notice the fuzziness of here presented environmental standards. The fuzziness is expressed by combination of crisp and nominal provisions in standards, as well as empty spaced cells in the provisions of standards from the tables. Also, the inherent uncertainty in crisp representation of standards has not to be forgotten.

The idea of fuzzy representation of standards for pollutant regarding human health is shown in Figure 4, but only as a concept. The regions of membership values, according to demands in standard are also shown, except for the date of achieving the standard that also could be presented as fuzzy region. It could be seen that standards for human health (Table 2) are more precise (or less fuzzy) than for ecosystems and vegetation (Table 3). More fuzziness stays in the latter standard for ecosystems, where there is no description of allowed frequency of exceeding and tolerant levels. Fuzzy approach could therefore cover regions of the standard which are not otherwise specified.

Table 2. Limit and tolerant values of concentrations of pollutants in air regarding human health (from GRC, 2005).

Pollutant	Averaging time	Limit level of concentration (LVC) μg / m^3	Allowed frequency of exceeding LVC per year	Tolerant level (TL) μg / m^3	Numerical value of tolerant level for period 2006–2010	Date to achieve LVC
PM–10 (I-phase)	24 hr	50	35	75	75 ÷ 5	Dec. 31, 2014
NO_x	24 hr	80	7	120 (not to be exceeded more than 7 times per year)	120 ÷ 5	Dec. 31, 2014
SO_2	24 hr	125	3	–	–	–

Table 3. Limit values of concentrations of pollutants in air regarding ecosystems and vegetation (from GRC, 2005)

Pollutant	Averaging time	Limit level of concentration (LVC) μg / m^3	Allowed frequency of exceeding LVC	Tolerant level (TL) μg / m^3	Numerical value of tolerant level for period 2006–2010	Date to achieve LVC
PM–10	–					
NO_x	1 year	30	–	–	–	Dec. 31, 2010
SO_2	Calendar year and winter time	20	–	–	–	Dec. 31, 2010

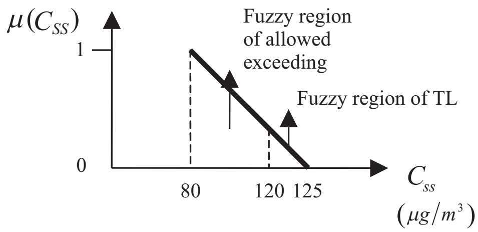

Figure 4. Linear fuzzy standard approximation for NO_x concentration from Table 2. regarding human health.

7. Conclusion

The variables of Eulerian air quality model, mixing height and wind velocity, are transformed from probability form of distribution to possibility form, latter could be understood as imprecise probability distributions. By the methods of fuzzy arithmetic, the solution of model equation in the form of fuzzy sets is found (9). The general relation (10) is proposed for construction of other fuzzy sets – linguistic variables for describing acceptability of interventions in the environment.

It is shown that air quality standards either for human health and ecosystems, yet trying to exploit the crisp form of representation, are mostly fuzzy in nature. They could be expressed as fuzzy sets and used in construction of linguistic variables.

The obtained linguistic variables will be considered as parts of hybrid models (combining mathematical and expert) in decision-making for air protection, specifically in environmental impact assessment.

Nomenclature

$a(\alpha)$, $b(\alpha)$	– α – cut at boundaries of interval for fuzzy number $\tilde{U}$
$c(\alpha)$, $d(\alpha)$	– α – cut at boundaries of interval for product of $W \cdot \tilde{H}_m$
$C_{i0}\left[\frac{M}{L^3}\right]$	– concentration of pollution substance entering the volume i
$C_i\left[\frac{M}{L^3}\right]$	– concentration in the volume i
$C_0\left[\frac{M}{L^3}\right]$	– concentration entering volume (box) in steady state

Eulerian model

$C_{ss}\left[\frac{M}{L^3}\right]$, here $\left[\frac{g}{m^3}\right]$	– steady state concentration
$\tilde{C}_{ss}$	– fuzzy number for variable C_{ss}
CS	– compliance side of the environmental standard
EIA	– environmental impact assessment
f	– function for transferring possibility values to membership (veristic) values, veristic function
H_m $[L]$, here $[m]$	– mixing height
$\tilde{H}_m$	– fuzzy number for variable H_m
L $[L]$, here $[m]$	– length of the box in the wind direction
LV	– linguistic variable
NCS	– non-compliance side of the environmental standard
$Q_m\left[\frac{M}{t}\right]$, here $\left[\frac{g}{s}\right]$	– total emission from the area sources
$q_i\left[\frac{M}{L^2 t}\right]$	– emission per unit area in volume i
p	– power-law exponent according to the class of stability of atmosphere
t $[t]$	– duration of continuous or semi continuous operation
$U\left[\frac{L}{t}\right]$, here $\left[\frac{m}{s}\right]$	– wind velocity

$U_{10}\left[\frac{L}{t}\right]$, here $\left[\frac{m}{s}\right]$	– wind velocity for $z = 10$ m
$U(z)\left[\frac{L}{t}\right]$, here $\left[\frac{m}{s}\right]$	– wind velocity depending on z
$\widetilde{U}$	– fuzzy number for variable u
x	– variable of possibility distribution
x_0	– mean value of variable x (according to frequency distribution)
$W\,[L]$, here $[m]$	– width of the box in Eulerian model (perpendicular to wind direction)
$z\,[L]$, here $[m]$	– height above the ground

Greek symbols

α	– alpha cut, generally
$\mu(.)$	– membership value of fuzzy set in interval [0,1], also possibility distribution in the same interval
$\mu(C_{ss})$	– possibility value of steady state concentration
$\mu(Q_m)$	– membership value of total emission
$\mu(x)$	– possibility distribution of variable x
σ^2	– variance obtained from frequency distribution
$\tau_r\,[t]$	– residence time

Mathematical symbols

$[C_{ss,1}(\alpha), C_{ss,2}(\alpha)]$	– α – cut at boundaries of interval of $\widetilde{C}_{ss}$
Δx	– change of length coordinate in wind direction for volume i

Subscripts

i	– belonging to volume i
0	– entering the volume in the model (both for differential and steady state)
ss	– steady state

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SAŽETAK

Neizrazito modeliranje u zaštiti zraka

Damir Rumenjak i Siniša Štambuk

Pokazuje se kako se klasično modeliranje kakvoće zraka iz površinskih izvora zamjenjuje modeliranjem na temelju neizrazitih skupova. Rezultati takvog modeliranja su neizraziti skupovi koji se mogu koristiti u tzv. hibridnim modelima za približno zaključivanje i podršku odlučivanju u zaštiti okoliša. Također se opisuje budući rad na konstrukciji lingvističkih varijabli i prikazivanju okolišnih normi neizrazitim skupovima u svrhu korištenja u hibridnim sustavima.

Ključne riječi: neizraziti skupovi, funkcije udjela, lingvističke varijable, modeliranje kakvoće zraka, eulerov volumni model, procjena utjecaja na okoliš, okolišne norme

Author's addresses:

Damir Rumenjak, Ministry of Environmental Protection, Physical Planning and Construction, Zagreb, Croatia, tel: +38516551390, e-mail:damir.rumenjak@zg.t-com.hr.

Siniša Štambuk, State Inspectorate, Zagreb, Croatia, tel: +38512451847, e-mail: sinisa.stambuk@zg.t-com.hr