Effects of earthquakes on buildings

Lecture given by
Professor Andrija Mohorovičić, Ph. D.

at the Croatian Society of Engineers and Architects (CSEA)
on March 1st, 1909

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Translated into English by Vjera Lopac

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Foreword to the English translation

The 1908 Messina (Italy) earthquake triggered intense activity among seismologists and engineers who started to seriously consider seismically resistant construction (see, for instance the paper by Sorentino, 2007). Andrija Mohorovičić was no exception. At the time he was a director of the Meteorological Observatory in Zagreb, but all of his scientific interests were already directed towards seismology (more on his life and work may be found in *e.g.* Skoko and Mokrović, 1982, 1998, or Herak and Herak, 2007). Although he was mostly interested in physical aspects of earthquakes and wave propagation (his greatest scientific accomplishment – proof of the existence of the crust-mantle boundary – is less than two years away), Mohorovičić was always deeply concerned with the engineering aspect of the problem. For instance, in his overview paper on the developments in seismology (Mohorovičić, 1913), he wrote:

> »… *Systematic study of earthquakes has also one very practical aspect. Strong earthquakes often cause great damage to houses and other buildings, and occasionally they level to the ground large and rich cities, and bury thousands of people under the ruins. Therefore, one of the most important goals of seismology is to theoretically study how the movement of the earth affects buildings, and to apply these results as well as the experience gained in catastrophic earthquakes to show the ways of constructing buildings resistant as much as possible against earthquakes. …*«

This topic was apparently a matter of great importance for him. On March 1, 1909, he held a lecture at the Croatian Society of Engineers and Architects (CSEA), in which he attempted to explain how the earth shakes during earthquakes, and what are the effects of this shaking on buildings. Later that year, in the newspaper article dedicated mostly to the effects of the Kupa valley earthquake of October 8 (the analyses of which led him to his epochal discovery), he describes damage to houses and other buildings, and adds:

> »… *I take this opportunity to warn all responsible institutions of outdated building codes, which completely disregard the ways earthquakes affect buildings. In March of this year I have delivered a lecture on that subject in the Society of Engineers and Architects in Zagreb, and I stressed the need to consider earthquakes when buildings are constructed. I will soon publish the extended edition of the lecture. So far, my appeals fell on deaf ears; after the lecture, many buildings were erected in Zagreb that pose threat for passers-by, as well as to themselves. …« (Mohorovičić, 1909a).*

Although – as he puts it – his was the voice of a man lost in the desert, there has been some response from authorities. The archives of the Department of Geophysics hold a letter (filed on April 18, 1910) from the Royal Department of Religion and Education in which he is asked to provide advice on how to construct schools, churches and parish houses to better resist earthquakes. Mohorovičić responded the same day, eagerly agreeing to act as consultant. He informed the Department that he has already finished the manuscript describing his studies (based on the above mentioned lecture), and urges the government to persuade the Department of Civil Engineering to cover the cost of publication. The letter ends with: »… *The book would be useful not only to the Royal Department of Civil Engineering, but also to the architects and entrepreneurs. It would promote improvement of the construction process, and would certainly stimulate many experts to keep considering this subject, and to use experience to perfect what the undersigned has started.«* On May 5, 1910 Mohorovičić despatched a 23-page document addressed to the Division of Internal Affairs and its Department of Civil Engineering, in which he put forward a remarkably detailed plan of actions leading to improvement of the building and construction in order to secure seismic resistance of edifices. As far as we know, the Government did not react at all…

The extended version of the lecture of 1909 was eventually published in 1911 in 9 sequels in the News of the CSEA (Mohorovičić, 1911a), and was reprinted in a booklet later in the same year (Mohorovičić, 1911b). This study, regarded by the Croatian scientific community to be the origin of engineering seismology in Croatia, is unknown to non-Croatian speaking professionals. On the centennial of the lecture, the Editorial board of Geofizika decided to publish its translation into English, thus providing the international seismological and civil engineering community an insight into this, generally unrecognized, aspect of Mohorovičić's professional activity.

The study itself is typical of most of the Mohorovičić's opus – it is a thorough, multifaceted insight into the problem of aseismic building design, with special emphasis to building types most frequently found in Croatia of that time. Being a transcript of articles from a non-scientific publication, only one formal reference – Baratta (1910) – is found of previous work done by foreign researchers. Nevertheless, Mohorovičić refers to unspecified Omori's papers and other Japanese investigations in several places. That he was in large part inspired by the work of Japanese and British seismologists working in Japan, is also clear from another lecture he gave in the CSEA (Mohorovičić, 1909b) about the Japanese studies on earthquake resistant buildings, citing works by Omori (1901) and Kikuchi (1904).

The study has an Introduction where Mohorovičić presents his motivation, but also introduces concepts of seismic hazard and earthquake risk in such a way that 100 years later, there is little one could add or improve. He deals with inhomogeneous distribution of seismicity, distinguishes between buildings according to their purpose and expected life-span, and even gives financial and economical motivation for building earthquake resistant houses. To the best of our knowledge, this is the first time anyone (in Croatia) attempted to use earthquake statistics to estimate cumulative effects a building will have to withstand during its lifetime. Here he also warns against the practice of considering only static loads, completely disregarding external excitation (except, occasionally, the wind).

The Introduction is followed by paragraphs in which Mohorovičić presents basic physics of the shaking of the ground during earthquakes. He clearly identifies resonance of the buildings and earthquake waves as the primary threat to structures, and goes on to describe eigen- and forced vibrations of buildings.

In the main body of the paper Mohorovičić analyses most common building elements and building types and their resistance and response to earthquake shaking. His excellent physical background helped him to deal not only with the simplest structures (like columns), but also with rather complicated buildings types. The calculations enabled him to draw a number of rules to be observed in order to build earthquake-resistant buildings, which are presented in the concluding paragraph. One century later, the 15 Mohorovičić's rules are still valid and applicable, which is easy to confirm if one compares them with the basic principles of conceptual design as listed in Eurocode-8 (2004).

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Introduction

After each major earthquake, in the engineering circles one discusses the damage which affected the buildings during the catastrophe, and possible means of prevention against the damage. From these discussions, many ideas usually emerge on alterations which should be introduced into the traditional building ways. But such is the human nature that, as soon as the first fears are over the feeling of security prevails, and nobody considers any further changes; moreover, even those which have been accepted immediately after an earthquake get forgotten. Another strong earthquake is needed to remind people that the building techniques should be further developed and improved.

The recent Messinian catastrophe has awaken us all and made us think of whether our own buildings could resist an earthquake of the strength similar to the one which affected the south of Italy on December 28th 1908. Anyone who has carefully examined the pictures of destruction which affected the towns of Messina and Reggio would be convinced that the damage of a similar earthquake in Zagreb would be much smaller; nevertheless, it is certain that many mistakes are still being made and that damage could be quite large.

There are two sources of mistakes which would seriously endanger our buildings. The first come from ignoring the ways the earthquake affects the buildings; the other is the inadequate construction process.

The best building designs sometimes come into the hands of a builder who, as a result of the financial greed or unconscientious competition, performs his job quite poorly. It also happens that the building is made with the sole purpose to be sold immediately, with a profit made as high as possible. In these cases one does not take care of the quality of work, but the only aim is that a building as large as possible is built with expenses as low as possible. It is clear that such buildings cannot be strong enough to resist an earthquake; it is therefore upon the authorities to strictly ensure a highly solid building procedure for each building.

Many properly executed buildings are not safe against earthquakes, simply because their foundations were made without considering the earthquake hazard. Investigating the damage on buildings from some major earthquakes during recent years, I obtained results which confirm that some general rules can be formulated. Every contractor should take these rules into account when planning the buildings, especially the very tall ones, and the buildings designed in agreement with these rules would be almost completely safe against earthquakes. One cannot speak of the absolute safety, because it is not known how strong the strongest earthquake can be, and probably it is not even possible to construct the building which would not be damaged by an exceptionally strong earthquake.

Regarding the purpose the buildings are constructed for, one should distinguish between the monumental and the ordinary buildings. The first are erected with intention to have a long lifetime, and there is no economizing with the money or with the material. The goal is to leave such a building to future generations to be reminded of the nation, the city or the person who erected the monument in honour of his own riches, greatness or glory.

The normal buildings have the purpose that the capital invested into them may bring as much of the profit as possible. The calculation must take into account: 1. the invested funds and the costs of the maintenance, of the future big repairs, taxes and similar expenses, and any risks concerning the building; 2. the regular profits and the risks concerning these profits.

It is not my intention to discuss details of the costs and the profits associated with the building. I only wish to mention that many different hazards with which any proprietor is confronted include also the danger of earthquakes. The owner and the contractor have to take this danger into account, and include it into plans according to their means and to the building's expected lifetime. When a monumental building is constructed, one should be aware that in its long lifetime it will be subjected to some extremely strong – or even catastrophic – earthquakes, and it will have to be made strong enough to resist any earthquake. In the case of an ordinary building one asks how large is the probability that during the lifetime of the building one or more damaging earthquakes, or even a disastrous one, will occur. According to this probability the funds invested into the safety of the building will be more or less profitably returned.

Some regions are subjected to large earthquakes, in others only very weak earthquakes occur, or there are even such regions where the earthquake is an absolute rarity. The builder should therefore take account of the region in which the site is located. From the statistical data, collected by Professor Kišpatić, Ph. D., during last 25 years, it follows that the most prone to strong earthquakes are the Zagreb and Varaždin counties, especially the town of Zagreb and its surroundings. In the second group is the whole Primorje region, from Rijeka to Senj and Karlobag. The third group comprises the Požega county. The other regions of our homeland are more or less safe from strong earthquakes.

From the data collected in the past, one can determine the probability of strong earthquakes in Croatia and Slavonia. In the last 50 years 91 strong earthquakes occurred in our region. Of these 57 earthquakes were in the Zagreb and Varaždin counties, 20 in Primorje and 30 in the Požega county. On the mountain of Medvednica (Zagrebačka gora) there were 30 of them. It follows that in the surroundings of Mt. Medvednica, including the city of Zagreb, three strong earthquakes occur every five years.

In the last 25 years there were 3 to 4 earthquakes which induced damage, and a disastrous one in the year 1880 with great damage. In the more distant past, the earthquake of March 26th 1502 is mentioned, when the tower of St. Mark's collapsed, the other one of the year 1590 when the Medvedgrad Castle was destroyed, of 1699 when again the tower of St. Mark's was destroyed, and another one of July 1st 1756. It is hard to estimate whether these earthquakes were stronger or weaker than the 1880 earthquake. These data show that in Zagreb we have one very strong earthquake each 100 years, one weaker but damaging earthquake every 10 years, and 3 moderate earthquakes within every 5 years.

It is assumed that an ordinary building has a duration of 150 to 200 years. During its lifetime this building will have to withstand one very strong earthquake, about 100 moderate earthquakes, 100 weaker earthquakes and 1500 to 2000 very weak earthquakes, whose origin lies below the town of Zagreb or very near to it.

If we add those earthquakes which have their foci in the greater Zagreb area, and whose shaking is also felt in Zagreb, then the numbers we obtain are very large.

It follows that the danger of earthquakes must be carefuly considered when building houses in Zagreb, and that it is profitable to pay more in order to make the building as safe against the earthquake as possible.

The safer our buildings will be, the lower will be the credit interests for such buildings, and the town will grow and develop faster if the foreign guests will be free of fear of earthquakes.

The building codes for the town of Zagreb contain several rules having the purpose to secure the building from the earthquake hazards. Nevertheless, there are still many mistakes in the ways of constructing the buildings which can be avoided without increasing the costs; there are some which could be easily avoided with only slightly increased costs, taking into account also the purpose the building will serve, and the durability it was designed for.

I decided to use these lines to explain to the highly esteemed gentlemen builders and contractors about the ways how the earth is trembling and how this trembling affects the buildings, and to stress some principles which every architect and every contractor should have in mind when building our houses.

In Europe, relatively little attention had been paid to the danger of earthquakes until now, and even today only the calculations of static loads are performed. Only for towers and high chimneys, the action of the wind is being taken into account. In Japan in the last 15 years many investigations of the action of earthquakes on buildings were done, and the results found there make the starting point for my discussion.

Before beginning the examination of the effects of earthquakes on buildings, one should have a precise idea on the ways how the earth can shake below buildings, and then to account for forces exerted by these tremblings. Then one should examine in detail how these forces affect the building as a whole and its separate parts.

A building is a very complex object, and the action of an earthquake on it is very complicated. Normally the buildings have the windows and the doors one above the other. The part of the wall standing between the windows or doors is the column. Buildings are normally made of certain number of such columns, which are carrying the loads of the floors and of the roof, and are mutually connected between the windows, i.e. at the floor levels, by using arches or long straight girders.

Building material also varies: it is mostly the wood (timber), brick, stone or the reinforced concrete, but there are buildings built with the combination of materials. Certain floors and the roof can be made of timber or of the reinforced concrete, but also using some mixture of various building material.

I. How the earth shakes and what are the forces exerted by shaking

§ 1. Investigation of earthquakes with modern instruments has given the following results on the ways how the earth shakes:

1. An earthquake consists of a series of periodic displacements of the earth, after which every point of the surface either returns to its initial position, or acquires a new position, corresponding to some linear displacement.

2. A sizeable linear displacement can be detected after an earthquake only by means of a very precise triangulation, but is often easily seen during large earthquakes, either as cracks appearing on the earth surface, or as a larger or smaller denivelation of the ground.

3. The periodic motion can be described as a sum of waves or oscillations in the three mutually perpendicular directions: one vertical and two horizontal directions, e.g. NS (north-south) and EW (east-west). If one combines the two horizontal directions into one resultant, one can talk about only one horizontal and one vertical component of the wave motion or oscillation of the earth.

§ 2.

Since the linear movements of the earth are either harmless or induce damage which can be neither predicted nor calculated, here we consider only the oscillatory or the wave motions.

A point performs a vibration when it first moves in some positive direction, for example towards the right hand side, and then reaches a certain largest distance with respect to its initial position. From there it returns, going in the negative direction, passing through its initial position down to the same maximal distance on the other side; after that it returns again and comes back to the initial point. The point A is moved first to a, goes back to A, continues until a_1, and returns to A. If there were no obstacles, this process would be continued endlessly.

If a certain point A (Fig. 1.) in the earth or on the surface of the earth acquires from the earthquake some velocity in the direction Aa, it shall be able to move in this direction only to the point where the elasticity of the earth absorbs the whole energy of its motion. Thus the motion from A to a is retarded,

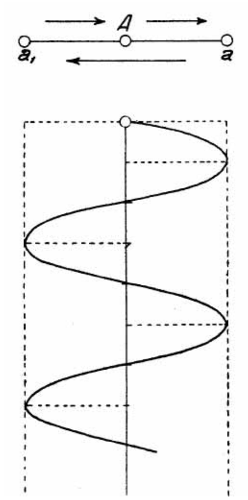

Figure 1.

or in other words: *in each position of the point A, which is not its initial position, a force is acting on the point oriented towards the initial position, and the acceleration of this point increases as the distance from the initial position grows*. For very small displacements Aa one can assume that the acceleration is proportional to the distance Aa.

The largest distance reached by the point, with respect to its initial position, is called the amplitude of the oscillation. The time needed for the point to perform the complete motion from A to a and back, passing through A to a_1 and then back to A, is called the period of oscillation.

If some point on the surface of the earth rises, it pulls with it all the surrounding points, so that they move in the same manner as the original point, but with a certain delay. From these points the movement is conveyed to further neighbouring points, etc. After some time the surface of the earth looks just like a surface of the water a short time after a stone has fallen in it, i.e. the waves are formed which, starting from the point at which the motion began, spread in all directions. Therefore this kind of oscillatory motion is also called the wave motion.

If the point A is at the moment t at the distance s from the initial position, the acceleration $\frac{d^2 s}{dt^2}$ arises, directed towards the initial position and proportional to the distance s. If one denotes with k the factor of proportionality, one has:

$$\frac{d^2 s}{dt^2} = -ks \qquad (1)$$

Hence follows the equation of motion of the point:

$$s = a \sin 2\pi \frac{t}{T} \qquad (2)$$

It follows from this equation that at the time $t = T/4$ the point is at the largest distance from the initial position i.e. $s = a$; at the time $t = T/2$ it is at the distance $s = 0$; at the time $t = 3T/4$ it is at the position $s = -a$, i.e. at the distance a on the opposite side; at the time $t = T$ it is again at the position $s = 0$. Thus T is the period of motion of the point and a its amplitude. The equation (2) hence represents the oscillatory motion.

Differentiating twice the equation (2), one obtains the value of acceleration:

$$\frac{d^2s}{dt^2} = -\frac{4\pi^2}{T^2} a \sin 2\pi \frac{t}{T} \tag{3}$$

or, comparing with equation (2),

$$\frac{d^2s}{dt^2} = -\frac{4\pi^2}{T^2} s \tag{4}$$

Comparison with equation (1) gives for the factor of proportionality $k = \frac{4\pi^2}{T^2}$.

It follows from equation (3) that the acceleration with which the earth acts on the moving point A at the initial point, i.e. at the time $t = 0$, is

$$\frac{d^2s}{dt^2} = 0$$

and that with increasing t it increases to the value

$$\frac{d^2s}{dt^2} = \frac{4\pi^2}{T^2} a.$$

At the time $t = T/2$, when the point has returned to its initial position, the acceleration drops again to

$$\frac{d^2s}{dt^2} = 0.$$

At the time $t = 3T/4$ the acceleration increases again and takes on the value

$$\frac{d^2s}{dt^2} = \frac{4\pi^2}{T^2} a$$

i.e. the same value as it had at the distance a on the other side, but this time in the opposite direction.

Thus, during one oscillation the acceleration increases twice to its maximal value, but at the first turning point the direction of the acceleration is opposite to its direction at the second turning point.

The value of the amplitude is determined by the velocity of the point at the moment when the motion started.

By differentiating the equation of motion (2) one obtains

$$\frac{ds}{dt} = \frac{2\pi}{T} a \cos 2\pi \frac{t}{T}.$$

Here $\frac{\mathrm{d}s}{\mathrm{d}t}$ represents the velocity of motion for an arbitrary position of the point, if the velocity at the initial time is known.

If at the time $t = 0$ the velocity is $\frac{\mathrm{d}s}{\mathrm{d}t}$=V, then

$$V = \frac{2\pi}{T} a \tag{5}$$

and hence $a = \frac{TV}{2\pi}$.

If the earth starts vibrating below an inelastic body, lying on its surface and fixed to it, with an amplitude a, the body will be moving together with the earth. At the beginning it obtains the velocity V. If the mass of the body is M, then the energy of the body at the initial time is $MV^2/2$, or, due to the equation (4), it is: $\frac{M}{2}\frac{4\pi^2}{T^2}a^2$. This energy is completely spent during the motion along the path a. If one substitutes the varying resistance of the earth with a constant force P, which acts unchanged on the whole path, and if one requires that this force along the path a exerts the work spending the whole energy of the body, it will be

$\frac{M}{2}\frac{4\pi^2}{T^2}a^2 = Pa$, and hence $P = \frac{M}{2}\frac{4\pi^2}{T^2}a$,

i.e. the work would be equal to the work done by a body with mass $M/2$ submitted to an unchanging maximal acceleration $\frac{4\pi^2}{T^2}a$.

§ 3.

If an arbitrarily tall elastic body (a building or a tower, etc.) is connected to the earth and stands vertically on it, and if we assume for simplicity that all its mass is concentrated in a point at the height l above the earth, then when investigating the motion of its centre of gravity we also must take into account the elasticity of the body.

If the centre of gravity is displaced from the point A (Fig. 2.) to a point B, while at the same time the base of the body remains at rest, its elastic bond with the earth would be bent, and the force induced in this way would push the centre of gravity of the body towards the point A. The body would start oscillating with the amplitude AB, provided that there is no obstacle hindering the oscillation.

The period T_0 of oscillation depends on the maximal acceleration attained at the point B i.e. on the elasticity of the element binding the body to the earth or to the base of the building. This motion of the body, which depends only on its own elasticity, will be called eigen or natural oscillation of the body.

If the body, fastened to the earth by means of an elastic bond, is at rest, and then the earth begins to oscillate, during some short time after the beginning of the motion the body will remain at rest, but its bond with the earth will be bent (Fig. 3). When the base moves from O to O_1, the body A is again at rest, but now the elastic force starts acting, and the body starts oscillating around the point A_1. For better clarity, the distance OO_1 is in the picture shown quite large, but in fact the body starts oscillating as soon as the base is displaced even a smallest length from the point O. In the next moment, the base get displaced from O_1 to O, whereas the body is moved from A_1 to A_2, and is now oscillating around the point A_3; after that the motion continues in the same manner.

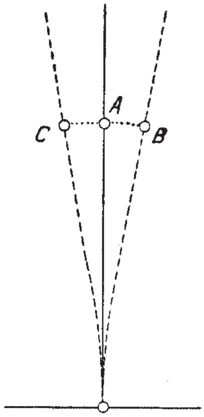

Figure 2.

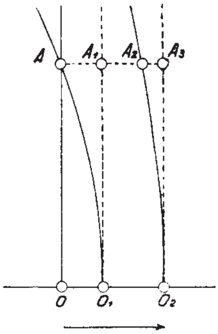

Figure 3.

The motion of the base corresponds to the velocity of the motion of the earth, but the motion of the centre of gravity depends both on the movements of the earth and on the elastic bond between the body and the earth. The motion of the centre of gravity thus consists of two parts: one corresponds to the movements of the earth, the other corresponds to the natural oscillations of the centre of gravity.

The induced natural motion is identical to the sound resonance. Thus, in all elastic bodies fixed to the earth, an earthquake induces natural oscillations, in other words, elastic bodies resonate with the earthquake. This phenomenon is of great importance when we study the damage to buildings caused by an earthquake, and should be theoretically investigated in more detail.

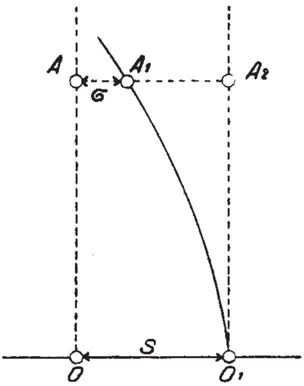

Figure 4.

Since this is a very complicated phenomenon, we first consider the simplest possible case. As the earthquake starts, a very weak natural motion begins, which gets stronger and stronger during the earthquake until it attains its highest amplitude. We shall consider only this final stationary state, and will derive how large is the part of the motion corresponding to the period of the motion of the earth, and how large is the other part, corresponding to the period of the natural oscillations. We denote the period of the earth motion by T, and the period of the natural oscillations of the body as T_0.

If the earth is, at a certain time, positioned at O_1 (Fig. 4.) i.e. at the distance s from the initial position, and the body in A_1 is the distance σ away from its initial position, then there is a force acting on this body directed to A_2, having the acceleration which depends on the period of the natural motion T_0 of that body and on its distance A_1A_2 from the new point of equilibrium at A_2. Analogously to the equation (1) or (4), the equation of motion of the body will be:

$$\frac{d^2\sigma}{dt^2} = \frac{4\pi^2}{T_0^2}(s-\sigma) \tag{6}$$

where the distance s corresponds to the position of the earth at the time t. If the period of motion of the earth is T and its amplitude A, we have $s = A \sin 2\pi \frac{t}{T}$.

In this way equation (6) becomes:

$$\frac{d^2\sigma}{dt^2} = \frac{4\pi^2}{T_0^2} A \sin 2\pi \frac{t}{T} - \frac{4\pi^2}{T_0^2}\sigma \tag{7}$$

The total integral of this equation is very complicated even for the stationary state, since, besides the basic oscillation due to the proper elasticity, higher harmonics also appear.

Presently we take into account only the fundamental natural oscillation. Later on, when we start investigating the effects of the earthquake on buildings, we shall have ample opportunity to include also the higher harmonics into our analysis.

The simplest integral of the equation (7) is

$$\sigma = a \sin 2\pi \frac{t}{T} + b \sin 2\pi \frac{t}{T_0} \tag{8}$$

where a is the amplitude of the oscillation with the period T equal to the period of the oscillation of the earth, and b is the amplitude of the natural vibration of the elastic body oscillating with the period T_0. The first of these two oscillations is called forced oscillation, and the second is the natural oscillation of the body. To find the amplitudes a and b, we must differentiate the equation (8) with respect to time t

$$\frac{d\sigma}{dt} = \frac{2\pi}{T} a \cos 2\pi \frac{t}{T} + \frac{2\pi}{T_0} b \cos 2\pi \frac{t}{T_0}.$$

At the beginning of the motion, when the earth just starts moving below the elastic body, it is at rest, i.e. for $t = 0$ we have $\frac{\mathrm{d}\sigma}{\mathrm{d}t} = 0$, thus

$$\frac{2\pi}{T} a + \frac{2\pi}{T_0} b = 0.$$

Hence it follows $b = -a \frac{T_0}{T}$[1], or, if we denote $\frac{T}{T_0} = n$, we obtain $b = -\frac{a}{n}$.

The amplitude b of the natural motion is proportional to the amplitude of the forced motion. To find the amplitude of the forced motion, let us differentiate the equation (8) twice and substitute the value obtained for $\frac{d^2\sigma}{dt^2}$ and σ into equation (7). After some reduction one obtains:

$$-\frac{a}{T^2} = \frac{A}{T_0^2} - \frac{a}{T_0^2}$$

and hence $a = \dfrac{A\dfrac{T^2}{T_0^2}}{\dfrac{T^2}{T_0^2} - 1}$ or $a = \dfrac{An^2}{n^2 - 1}$, thus $b = -\dfrac{An}{n^2 - 1}$.

The equation (8) becomes

$$\sigma = \frac{An^2}{n^2 - 1} \sin 2\pi \frac{t}{T} - \frac{An}{n^2 - 1} \sin 2\pi \frac{t}{T_0}\ , \tag{9}$$

which is the simplest solution of the problem, but it is sufficient for our purpose.

As the period of the natural motion of the elastic body and the period of the earthquake can have various values, so will also the amplitudes a and b attain a range of values. One can distinguish between two main cases:

1) the period of the earth is smaller i.e. shorter than the period of the natural motion: $T < T_0$, thus $n < 1$. For that case the equation (9) can be written as

$$\sigma = -\frac{An^2}{1 - n^2} \sin 2\pi \frac{t}{T} + \frac{An}{1 - n^2} \sin 2\pi \frac{t}{T_0}$$

[1] Typographic error in the original: $b = -T_0 / T$ (MH)

i.e. at the beginning, the forced motion has the direction opposite to the direction of the motion of the earth, and the natural motion has the same direction as the earthquake.

If n is very small, i.e. the motion of the earth is very rapid in comparison with the natural motion of the body, for the limiting value $n = 0$ it will be: $a = -\frac{An^2}{1-n^2} = 0$, and also $b = \frac{An}{1-n^2} = 0$, i.e. the earth is moving below the body, while it remains at is place. The bond of the body with the earth is swaying back and forth in such a way as to keep the centre of gravity at rest.

If a building as a whole has a very large period of the natural oscillation in comparison with the period of the earthquake, its centre of gravity will oscillate relatively to the earth (or foundation), but in the opposite direction. Somebody who would be in the cellar at that moment would detect the earthquake such as it is, the one who would be at the centre of gravity would not detect anything, while the person on higher storeys would also sense a strong earthquake, but in the opposite direction. Since there is no ideal building, the person in the centre of mass would also detect a weak earthquake.

With increasing n increase also the amplitudes of both forced and natural oscillations, until, in the limiting case $n = 1$, both a and b become extremely large. If by any chance the period of the earthquake and the period of the natural motion become equal, the motion becomes extremely strong. In reality the motion is never infinitely large, due to the friction, and also simply because the equation is valid only for small amplitudes, in comparison with the dimensions of the body. But even if we were able to include the friction into the calculation, the amplitude would be found to grow to very large values.

The period of oscillation of the earth in local earthquakes is quite small (0.5 to 2 seconds). The building with very quick natural motions will greatly suffer by these local earthquakes, because it will strongly oscillate itself. The periods from remote earthquakes are large, i.e. the oscillation is slow, and will excite into motion those buildings which have slow natural motions. Since the amplitude of the motion of the earth diminishes with the distance from the earthquake focus, generally the buildings with slow oscillations will be less damaged than those oscillating quickly.

If the period of the motion of the earth is larger than the period of the natural motion, i.e. if $T > T_0$ or $n > 1$, then the equation (9) acquires the form

$$\sigma = \frac{An^2}{n^2-1}\sin 2\pi\frac{t}{T} - \frac{An}{n^2-1}\sin 2\pi\frac{t}{T_0}.$$

The forced motion has the same direction as the motion of the earth, contrary to the wavy motion, whose direction at the beginning is opposite.

With increasing n, the ratio $\dfrac{n^2}{n^2-1}$ is approaching the value 1, and for $n \quad \infty$ the exact value of the amplitude would be $a{=}A$. At the same time, b would be diminishing with n, so that for $n = \infty$ $b = 0$.

The next table shows the amplitudes a and b for some important values of n. All these amplitudes have been calculated for the amplitude of the motion of the earth A = 1. Underscored values indicate that the initial motion of the body (i.e. at t = 0) has direction opposite to the direction of the motion of the earth.

n =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
a =	$\underline{0.01}$	$\underline{0.04}$	$\underline{0.10}$	0.19	$\underline{0.33}$	$\underline{0.56}$	$\underline{0.96}$	$\underline{1.78}$	$\underline{4.3}$	∞	5.8	3.3	2.4
b =	0.10	0.22	0.33	0.48	0.67	0.94	1.37	2.22	4.7	∞	5.2	2.7	1.9

n =	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	20.0
a =	2.0	1.8	1.6	1.4	1.3	1.2	1.1	1.1	1.0	1.0	1.0	1.0	1.0
b =	$\underline{1.5}$	$\underline{1.2}$	$\underline{1.0}$	$\underline{0.8}$	$\underline{0.67}$	$\underline{0.5}$	$\underline{0.38}$	0.27	0.21	0.17	0.13	0.10	0.08

Figures 5a—5f depict the motion of the body separated into components: the natural motion ········, the forced motion — — — and the motion of the earth ———. On the same picture the resultant of these three motions, with respect to the building base, is also shown as ×××××××.

This relative motion can be calculated if one adds up the forced and the natural motions, and then subtracts from the sum the motion of the earth. It is quite clear from these pictures that the bending of the elastic body increases as n approaches unity, i.e. as the period of the natural motion becomes closer to the period of the earthquake oscillations.

For practical purposes it is important to understand that the motion of the centre of gravity is actually described with respect to the base, rather than to the motionless earth. This allows us to determine what is the displacement of the centre of gravity from the vertical line above the base, i.e. how large is the bending of the elastic body. If we denote by y the distance from the centre of gravity to the vertical above the base, by σ the distance of the centre of gravity from the resting position of the earth, and by s the distance between the base and the position of the earth at rest, we have:

$$y=\sigma-s.$$

Substituting the known values for σ and s, after some necessary reduction we obtain

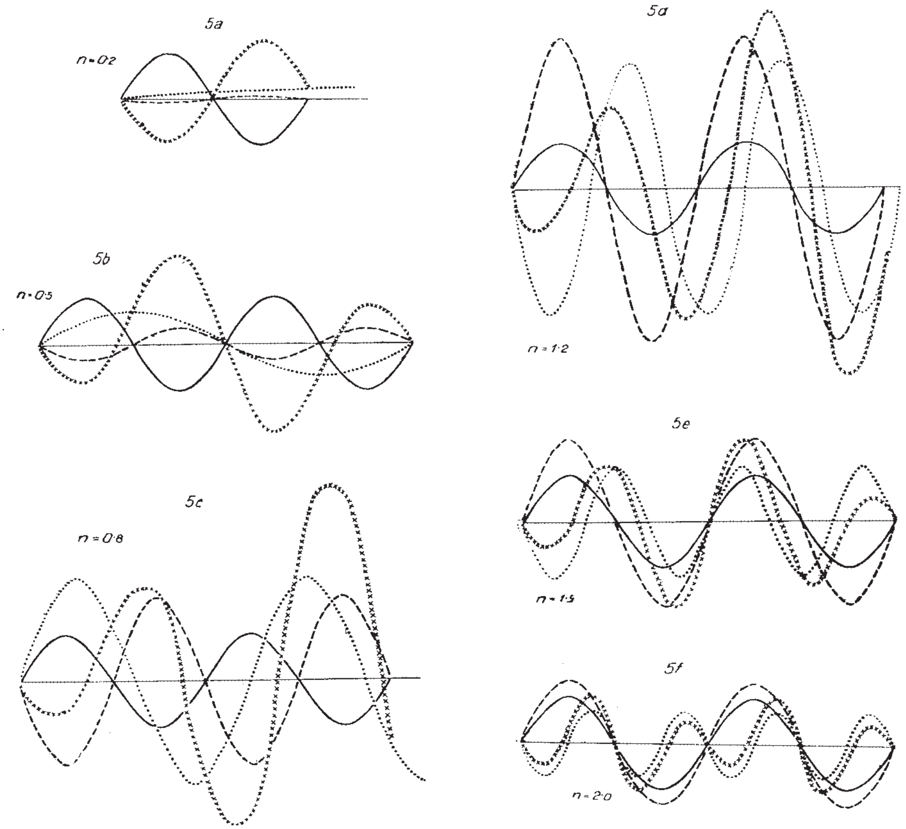

Figure 5.

$$y = \frac{A}{n^2 - 1} \sin 2\pi \frac{t}{T} - \frac{An}{n^2 - 1} \sin 2\pi \frac{t}{T_0}.$$

For $n < 1$ it will be:

$$y = -\frac{A}{1 - n^2} \sin 2\pi \frac{t}{T} + \frac{An}{1 - n^2} \sin 2\pi \frac{t}{T_0}.$$

If $n < 1$, i.e. if the velocity of the oscillation of the earth (or earthquake) is larger then the velocity of the oscillation of the building, then for the extreme case $n = 0$ the amplitude of the forced oscillation will be equal to A, i.e. equal to the amplitude of the earthquake, whereas the direction of the motion of the body will be opposite to the direction of the motion of the earth. With increasing n, the amplitude of the motion of the centre of gravity will attain extremely large values for $n = 1$. If $n > 1$, when n grows farther, the amplitude of the forced motion decreases, and for very large values of n it vanishes. The amplitude of the natural oscillation attains very large values when $n = 1$, but drops to small values both for large and for very small values of n.

If we denote, as in the previous example, the amplitude of the forced motion as a and the amplitude of the natural motion as b, the amplitude of the forced motion will be:

for $A = 1$

n =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a_1 =	1.010	1.042	1.099	1.191	1.333	1.563	1.961	2.778	5.263	∞
n =	1.2	1.5	2.0	3.0	4.0	5.0	6.0	8.0	10.0	20.0
a_1 =	2.273	0.800	0.333	0.125	0.067	0.042	0.029	0.016	0.010	0.003

The amplitude of the natural motion will be given later.

The amplitude of the resultant of both natural and forced motion will be slightly smaller than the sum of the two amplitudes, but the difference is not large. Therefore one can use the sum of the two amplitudes as the resulting amplitude. In the same way, the maximum resultant of the accelerations could be taken as equal to the sum of the maximal accelerations of the two components, or:

$$\alpha_{\max} = \frac{4\pi^2}{T^2}\frac{A}{n^2-1} + \frac{4\pi^2}{T_0^2}\frac{An}{n^2-1}$$

Recalling $\frac{T}{T_0}=n$, this can be reduced to the form

$$\alpha_{\max} = \frac{4\pi^2}{T^2} A \left(\frac{n^3+1}{n^2-1}\right) \text{ for } n>1, \text{ or}$$

Recalling $\frac{T}{T_0}=n$, this can be reduced to the form

$$\alpha_{\max} = \frac{4\pi^2}{T^2} A \left(\frac{n^3+1}{n^2-1}\right), \text{ for } n>1, \text{ or}$$

$$\alpha_{\max} = \frac{4\pi^2}{T^2} A \left(\frac{n^2}{n-1} - 1\right);$$

or

$$\alpha_{\max} = \frac{4\pi^2}{T^2} A \frac{1+n^3}{1-n^2}, \text{ for } n<1,$$

or

$$\alpha_{\max} = \frac{4\pi^2}{T^2} A \left(\frac{n^2}{n-1} + 1 \right).$$

The next table presents maximal accelerations of the total motion, starting with T = 0.1 to 5 seconds and for n from 0.1 to 2.0.

Since we will later use this acceleration, and will need just the expression $\frac{\alpha_{\max}}{g}$, we list in the table also the accelerations expressed as parts of the acceleration of gravity. The value of 1 mm is taken for A.

From this table we see that effects of the earthquake on buildings get smaller when the period of the earth oscillations increases. When the period attains the value of 2 seconds, the effects are so small that one can consider the waves with amplitude larger than 2 seconds harmless, if their amplitude is not exceptionally large.

With increasing n increases also the action of the earthquake, and for n = 1 it is exceptionally strong. For n larger than 1 the effects of the earthquake are decreasing, but for values of n above n = 2 they slowly increase again.

The total energy of the forced and natural motions of the body is larger than the energy of the earth, because whereas the maximal energy of the earthquake equals

$$\frac{4\pi^2}{T^2} A,$$

the maximal energy of the elastic body is somewhat smaller than

$$\frac{4\pi^2}{T^2} A \frac{1+n^2}{1-n^2}.$$

The last expression is always larger than the former one.

Table for $\dfrac{\alpha_{max}}{g}$

				n			
T	0.05	0.1	0.2	0.3	0.4	0.6	0.8
0.1	–	0.407	0.423	0.455	0.510	0.765	1.692
0.2	–	0.102	0.106	0.114	0.128	0.191	0.423
0.3	–	0.045	0.047	0.051	0.057	0.085	0.188
0.4	–	0.025	0.027	0.029	0.032	0.048	0.106
0.5	–	0.016	0.017	0.018	0.020	0.031	0.068
0.6	–	0.011	0.01	0.013	0.014	0.021	0.047
0.7	–	0.008	0.009	0.009	0.010	0.016	0.034
0.8	–	0.006	0.007	0.007	0.008	0.012	0.027
0.9	–	0.005	0.005	0.006	0.006	0.010	0.021
1.0	–	0.004	0.004	0.005	0.005	0.008	0.017
1.2	–	0.003	0.003	0.004	0.004	0.005	0.012
1.5	–	0.002	0.002	0.002	0.002	0.003	0.008
2.0	–	0.001	0.001	0.001	0.001	0.002	0.004

				n			
T	0.9	1.2	1.5	2.0	3.0	5.0	10.0
0.1	3.666	2.497	–	–	–	–	–
0.2	0.916	0.624	0.353	0.302	–	–	–
0.3	0.408	0.278	0.157	0.134	0.157	–	–
0.4	0.229	0.156	0.088	0.076	0.088	–	–
0.5	0.147	0.100	0.056	0.048	0.056	0.084	–
0.6	0.102	0.069	0.039	0.034	0.009	0.058	–
0.7	0.075	0.051	0.029	0.025	0.029	0.043	–
0.8	0.057	0.039	0.022	0.019	0.022	0.033	–
0.9	0.046	0.031	0.018	0.015	0.018	0.026	–
1.0	0.036	0.025	0.014	0.012	0.014	0.021	0.080
1.2	0.026	0.017	0.010	0.008	0.010	0.015	0.056
1.5	0.016	0.012	0.006	0.005	0.006	0.010	0.036
2.0	0.009	0.006	0.004	0.003	0.004	0.005	0.020

As mentioned earlier, a stationary state of the motion of the elastic body stabilizes only after several oscillations of the earth, during which time the acceleration increases to the maximal value.

This derivation is valid only for the case when the motion retains its direction and the same period for a longer time. However, during the earthquake both the period and the amplitude are continually changing, so that the stationary state is never attained. Therefore the maximal acceleration of the elastic body is always smaller than the maximal acceleration possible for the stationary state.

If an elastic body receives a kick by which it is launched into oscillation, its first oscillation will be the largest. Every next oscillation will be smaller, and after a certain number of oscillations the motion will stop. The reason of this should be sought in various obstacles which the body meets. These obstacles are called friction. Since for each subsequent oscillation the amplitude is smaller than for the previous one, the amplitude of the final stationary state, consequently also the maximal acceleration, will be considerably smaller than the one obtained theoretically without accounting for friction.

If an engineer takes as the basis of his calculations the maximal accelerations such as given in the previous table, he accounted for much more than was necessary. It can be said that his calculations account for a *k* times greater safety. Exactly how large is this safety coefficient is hard to state, since the friction differs from building to building.

§ 4.

For the vertical component of the motion of the earth the same conclusions are valid as for the horizontal component. The acceleration of the motion of the earth acts along the vertical, and alters its direction – at one time it is in the direction of gravity, at another it is opposite to it. If the acceleration of gravity is denoted as g (= 9800 mm/s^2) [2], then the motion of the building has the acceleration

$$\alpha = g - \frac{d^2\sigma}{dt^2}$$

assuming that we take the orientation towards the centre of the earth as positive. If, as in the simplest possible case, we have

$$\sigma = A \sin 2\pi \frac{t}{T}$$

and if, during the first displacement, the earth is going downwards, after one quarter of the full oscillation in the lowest position of the building it will be

$$\frac{d^2\sigma}{dt^2} = -\frac{4\pi^2}{T^2} A.$$

[2] In the original, Mohorovičić uses abbreviated notation and expresses acceleration as 9800 mm. (MH)

Thus the acceleration of the building will be

$$\alpha_1 = g + \frac{4\pi^2}{T^2} A$$

i.e. at that moment the building will be heavier, since if the mass of the building is M, its weight is at rest Mg, and in the lowest position it is by the amount

$$\frac{4\pi^2}{T^2} MA$$

larger than its normal weight. Thus, the earth is at that moment pushing on the building from below with the pressure $\frac{4\pi^2}{T^2} MA$.

When the earth rises together with the building to the highest position, the acceleration changes its direction and is

$$\alpha_2 = g - \frac{4\pi^2}{T^2} A,$$

i.e. the building is lighter by $\frac{4\pi^2}{T^2} MA$.

In this way the vertical component of the motion of the earth acts as the periodic compressing and stretching of the building in the vertical direction.

If, for some value of the period T, the amplitude A would be able to increase so that $g = \frac{4\pi^2}{T^2} A$, then at its highest position the building would loose its whole weight, i.e. the vertical pressure of certain parts to the other parts of the building would vanish completely.

For a further increase of the amplitude, α would become negative and the building would bounce from the earth up in the air.

The next table shows the amplitude for which would be, for certain periods, $\frac{4\pi^2}{T^2} A = g$, with $g = 9800$ mm/s^2.

T/s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	2.0
A/mm	2.5	9.9	22.4	39.7	62.1	89.4	121.7	158.9	201.1	248.3	993.2

The amplitude of the earthquake, needed if objects should be bouncing of the earth, increases with the square of the period, i.e. for very short periods a small amplitude is necessary for the bouncing, but with the increasing period the amplitude should steeply grow. For a period of 0.1 second it would be sufficient that the amplitude exceeds 2.5 mm, in order that the objects on the earth surface bounce from it. At the period of 1 second, this amplitude should be 25 cm, and for the period of 2 seconds, the amplitude should be approximately 1 meter.

Just as the horizontal motion of the earth provokes the bending of any vertically oriented elastic body on the surface of the earth, so the vertical component of the motion causes the longitudinal motion of all vertically extended objects, and a transversal motion of all horizontally extended objects, supported on one or both ends. In both cases one forced motion and one natural motion of the body exist, and both oscillations obey the same rules which were valid for the bending.

As was already mentioned in § 1, an earthquake is composed of a series of wavy horizontal and vertical oscillations. The horizontal motion constantly changes both its direction and its amplitude. When in an everyday conversation one hears talk of the direction of the earthquake, what is usually meant is either the prevailing direction of the oscillation or the direction of the strongest oscillations.

The amplitude of the motion of the earth can assume different values. For the largest catastrophic earthquakes it can be 5 cm or more, but usually it is very small.

Duration of the earthquakes can also vary considerably. Local earthquakes, i.e. those which have the focus directly below the shaken place or in the close vicinity, normally are short, from few seconds to several minutes, but sometimes also much longer. Further from the place the focus of the earthquake is, longer is the duration of the earthquake; an earthquake from a distant source can cause trembling of the earth for hours.

The difference should be stressed between the true duration of an earthquake, which only a seismograph can establish, from the duration sensed by people. People sense stronger shaking for shorter oscillation periods. When the period is between 0.3 and 0.6 seconds, people can detect an earthquake if the amplitude is 0.01 s[3]. If the period exceeds 1 second, the trembling is felt only for the strong earthquakes. So it can happen that a person can feel vibrations for the few first seconds during a weak earthquake, and for only first 15 to 30 seconds during a strong one. In fact, the earth is oscillating much longer, 1 to 3 minutes during very weak earthquakes, 10 minutes with stronger ones, and a quarter of an hour or more during very strong earthquakes.

The period of the motion of the earth is also variable. It is very low for the local earthquakes, whereas during distant earthquakes the incoming waves can have various periods and amplitudes.

A local earthquake usually starts with one or several very strong short period oscillations. They are followed by a series of very strong oscillations. After

[3] This is a typographic error in the original. It should probably be mm. (MH)

that the trembling becomes weaker and weaker, and the period increases. According to Omori, the period of very strong local earthquakes is about one second, and for a disastrous earthquake it is one to two seconds. The weaker earthquakes have much lower periods. In very strong earthquakes, however, the strong oscillations with period of one or two seconds are mixed with short waves whose amplitudes are much smaller. These rapidly oscillating waves of short periods are the most dangerous, since they have very large accelerations. They can cause great damage, especially if any one among them has an extremely short period.

For local earthquakes, the vertical oscillation of the earth is usually weaker than the horizontal motion. Alone it will cause no great damage, but in combination with the horizontal motion, it can be very dangerous.

During distant earthquakes different types of waves arrive. Three main phases are regularly distinguished for distant earthquakes. First arrive the waves along a tilted straight line coming through the earth from the earthquake source, and they cause both horizontal and vertical motion with short periods of 3 to 5 seconds. After them arrives the second phase with slower waves[4], and then the third phase which starts with very slow waves, often with periods of 30 to 60 seconds, culminates with largest waves having different periods, and ends with slow waves, which become weaker and weaker, until the motion completely stops.

Due to large accelerations and sizeable distortions they often cause to the buildings, the dangerous ones are only the rapid[5] waves, thus only local or very near earthquakes. Distant earthquakes do not cause damage, except maybe in some very old and shabby structures, whose period of natural oscillation, because of defective internal bonding, has become very long.

Until now we have assumed that both horizontal and strictly vertical motions have very large wavelengths, so that there is no denivelation of the ground, i.e. there are no observable wave-like displacements of the earth. If for example, one has a very large vertical amplitude of 5 cm with the period of 1 second, then the waves 1 km long cause a denivelation of 10 cm on the length of 500 meters. The maximal denivelation will be 6 cm per 100 m. Such denivelation will never cause damage. But if the velocity of oscillations were 100 meters per second, then for the same amplitude of the vertical motion and the same period the denivelation would grow to 60 cm per 20 meters, causing the maximal inclination of the walls to be about 2°. Such abrupt denivelation can be disastrous. In many cases of earthquakes the wavy motion of the earth was observed, and the short waves cannot be excluded.

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4 By 'slow waves' Mohorovičić here means 'slowly oscillating', long-period waves. (MH)

5 'rapid' here means 'rapidly oscillating', short-period waves. (MH)

Earthquakes are not the only causes of vibrations of the earth: every motion of heavy objects on its surface can provoke the wavy motion. The gun projectiles, land-mines, explosions, the motion of the railway trains or heavy vehicles can induce the oscillations of the earth. Normally the amplitudes of such oscillations are extremely small, and the periods are short. In our neighbourhood, some 70 meters away from the observatory, there is a funicular, going up and down every few minutes. The period of the motion of the earth below the observatory is so short that it cannot be measured, but one estimates with certainty that it does not exceed 0.1 second, and that its amplitude is approximately 0.001 mm.

The winds can also cause rather strong wave motion of the earth.

All these vibrations, although weak, are repeated daily hundreds of times, with the consequence that even the most solid buildings gets weakened.

The wind causes large pressure on the buildings, and, as it blows in gusts with approximate periods of 5 to 15 seconds, it causes the same types of oscillations as the earthquake does: both forced motion and natural motion are observed. As the wind has periods much longer than the natural period of the building, the building will bend until the point where the maximal force produced by the elasticity of the building equals the force caused by the pressure of the wind. Therefore the building will be not only leaning but will be also swaying as a whole.

With very high and narrow buildings, such as the towers, the natural motion can become very strong, if the period of the wind coincides with the period of the eigen motion. If, for example, the top of a tower is displaced some 10 cm and then the wind suddenly stops, the tower will suddenly start moving towards the equilibrium position, due to its own elasticity. It will pass through the equilibrium position and start oscillating with the amplitude steadily growing to values higher than 10 cm. If the period of the natural motion of the tower top is small, the induced acceleration can be so large that the tower breaks. To diminish this acceleration, in the interior of the tower one can put a heavy weight hanging on a long wire, which will oppose the sudden movement of the tower top.

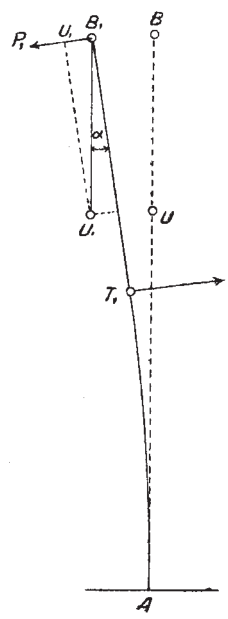

Figure 6.

If AB (Fig. 6) is the axis of a vertical elastic body, such as a tower, and in the point B a heavy weight U is suspended, the weight U will move to the position U_1 if the point B moves to B_1. The axis of the body bends into the shape AB_1. In this position the elastic force P acts on the centre of gravity

T_1 in direction T_1P, and the weight U acts with its component $U_1 = U \sin\alpha$ in direction opposite to the direction of P. In this way the force P, which tends to restore the axis into the equilibrium position, can be reduced at will, and one obtains a motion as slow as needed.

This procedure of damping is efficient only in the case when the motion of the point B has a period much larger than the period of the motion of the weight U, since if the period of the point B is smaller, then the weight practically stays at its initial position, while the point B moves to B_1. This method can be used for damping the motion only with the forces which slowly bend the elastic body. In the case of an earthquake such a weight is of no use.

Investigating the action of earthquakes on buildings, we have discussed the simplest case, where the whole building is constructed as a single and simple elastic body, whose whole mass is concentrated in the centre of gravity, and the centre of gravity is connected with the earth by means of an elastic bond having no weight. In reality, however, the buildings are very complicated structures, composed of a great number of elastic bodies. If the earth starts moving, the elastic part of the body which is directly connected with the earth, starts moving. This part pulls all the other parts connected with it and sets them to motion, and they act in the same fashion on the next neighbouring parts. Each of these bodies also acts back on the bodies which have caused its motion. This gives rise to a very complicated motion.

Moreover, if we know that each of these bodies exerts two types of motion: the natural one and the forced one, this makes the final motion even more complicated. The building will oscillate as a whole, but each of its parts will oscillate separately.

The general solution of the oscillation problem for such a complex body is not possible, but nevertheless we can apply the obtained results to the building as a whole and to its separate parts.

II. Action of earthquake on buildings

§ 5. Buildings are horizontal, vertical or, less often, oblique elastic loadbearers.

Materials of which they are made are steel, iron, concrete (common or reinforced), timber, stone and bricks, etc.

Small buildings can be made in one piece (monolith). Large buildings can be monoliths only if made of the reinforced concrete. The steel, iron and timber are used to construct bigger buildings, by binding smaller pieces together with nails or other kinds of fasteners. If the fasteners are of good quality, such a building can be regarded as a single piece.

When one builds with stones and bricks, the building blocks are simply laid one on the other and one by the other, while the space between them gets filled with the mortar. The purpose of the mortar is to fill the internal empty spaces and to glue parts to each other. The mortar can either be made of the simple clay or lime mixed with more or less sand, or of the cement mixed with sand. The clay or lime with sand are used just for filling the internal spaces. The lime with small admixture of sand and the cement will glue the separate parts together.

Very often the construction is performed by using mixed materials such as timber and bricks or iron and bricks. The ceilings of stone or brick houses are made either of wood, or the reinforced concrete, or of some other mixture of materials. They are placed on the main load-bearing walls and are linked with them more or less strongly by iron fasteners.

The building must be able to support both its own weight and the weight of the load put on it. Furthermore, it has to be strong enough to be able to resist the n times larger load than it is supposed to bear without any damage. Extremely high buildings must be able to withstand the strongest wind pressures, i.e. 150 to 250 kg per m^2 of the surface exposed to the wind.

If we want to make our building safe against earthquakes, we have to set up additional conditions. The action of an earthquake on the elastic body is proportional to the maximal acceleration of the earthquake and to the mass of the body. The fundamental equations of motion of the elastic body are valid only within the elastic limit. Therefore a solid building is required to be as light and as strong as possible, and to have a very high limit of elasticity. The timber conforms best to these requirements, since it has the highest strength in relation to its weight and possesses a very high limit of elasticity. Equally good is the steel, only it is heavier and more expensive. The iron has a much lower elastic limit. The concrete reinforced with iron or, even better, with steel, can be made as strong as needed, and its advantage in comparison with other materials is that the whole building can be executed as a single piece. Stone and bricks used with the ordinary mortar, prepared with a lot of sand, have almost no elasticity, but used with the mortar made with cement they can be sufficiently elastic.

If a building is made of one piece, i.e. of the reinforced concrete or of bricks or stones glued with a high quality cement, it can be considered as a single hollow column. If this is not the case, it can consist of columns linked together more or less strongly. Some ceilings are used both as the loadbearers and as bindings between the main columns from which the building is constructed.

Since every building can be assumed to be either a column or a set of columns, we shall at first discuss the action of the earthquake on a filled or a hollow column, and then we shall consider the building as a whole.

The forced motion

§ 6. Action of the earthquake on the column foundation

Imagine an upright monolith with the lower end buried in the earth (Fig. 7). Let the centre of gravity be at the point T and the centre of the action of the earthquake at the point S. If, during an earthquake, the earth oscillates together with the column around the equilibrium point O up to the turning points displaced from it by *a* mm, at the point O both the earth and the column have the maximal velocity. Along the distance from O to the remotest point, the resistance of the earth increases and eventually stops the column. At the largest distance on the right-hand side the resistance of the earth acts towards the left-hand side with the force $M\alpha$. If the force $M\alpha$ is needed for the earth to stop the motion of the column, then the same force but oppositely oriented $-M\alpha$ acts on the centre of gravity of the column. If we wish to know what is the action of the column relative to the earth at rest, we can oppose its motion using the force $-M\alpha$ which acts at the point S.

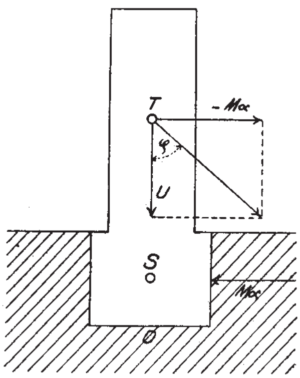

Figure 7.

The foundation of the column is pressing on the earth with the force $-M\alpha$ first from one and then from the other side. The forces $M\alpha$ at the point S and $-M\alpha$ at the point T act as a couple, tilt the column, and cause the soil around the base of the column to be moved apart.

The earth resists the horizontal pressure of the column partly by friction on the lower surface of the foundation and partly by the normal resistance on the left and right side of the foundation.

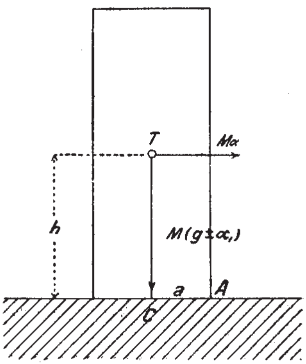

Figure 8.

The weight of the column U, together with the horizontal force $M\alpha$, at the point S will give the resultant which is inclined in respect to the base. Let the maximal acceleration of the vertical component of the earthquake be α_1, then the limiting weight of the column is equal to $U\left(1 \pm \frac{\alpha_1}{g}\right)$. The resultant of both forces, equal to $U\sqrt{\left(\frac{\alpha}{g}\right)^2 + \left(1 \pm \frac{\alpha_1}{g}\right)^2}$, closes with the vertical direction the angle φ, whose tangent is $\tan\varphi = \frac{\alpha}{g \pm \alpha_1}$.

During an earthquake, the foundation of the column, or of any building, compresses the soil alternately in the vertical and the oblique direction, to either the left-hand or the right-hand side. The maximal angle of incline is determined by the equation given above and is independent on the weight of the body.

For example, let us take the values of the accelerations as $\alpha=0.4g$ and $\alpha_1=0.2g$. The resultant of the compressive force will be between $0.9U$ and $1.26U$, while the angle φ will be between 18.5° and 26.5°.

This is the reason why it is not advisable to build heavy buildings on the soft and steep ground.

If the length between points S and T is ST = d, the torque tending to topple the column is equal to $dM\alpha$. The earth has to counteract with the same torque but having the opposite orientation.

If the soil around the foundation is soft, the foundation will broaden the pit in which lies the column and at the same time will widen its aperture. After the earthquake the column will stay tilted or, in the extreme case, will be torn from the pit and overturned. If the soil is hard, the foundations will remain unaffected.

§ 7. Connected columns

The columns are seldom made as one piece with the foundations. The foundation is usually a separate piece of material and the column is simply placed on the top of the foundation without any mortar, or it can be more or less strongly glued to it or fastened by bolts or rivets. The first case, when the foundation makes one piece with the ground, need not be considered separately, since the column in this case can be treated as simply posed on the ground. It follows that in the contact surface of the column placed on the ground or on the foundation acts a horizontal force $M\alpha$, which tends to push the column horizontally. This force, as explained above, together with the weight of the column, produces an inclined compressing force on the base plane, which grows when the maximal vertical acceleration of earthquake increases.

There has been no evidence in the past that the vertical acceleration of the earthquake would ever, on a large scale, attain the values close to the acceleration of gravity. Locally, however, it is not improbable, since it has been noticed many times that during strong earthquakes the objects have been bouncing from the earth.

Therefore, when the small columns are concerned, the angle typical for the force at the base level can be expected to be quite large and approach 90° or sometimes even exceed 90°.

When this angle becomes very large, there is the danger that the base of the column would start sliding. The weaker the connection between the column and the foundation, the larger will be the danger of sliding. Such a sliding has been observed during all strong earthquakes, especially with the tombstones and monuments.

Opposite to this horizontal force acts the friction between the lower surface of the column and the base. The points of application of these two forces, as a rule, do not lie on the line determined by the direction of the action of the earthquake. Therefore the two forces make a couple, causing the rotation of the column at its base. In this way one can explain the displacements of columns, chimneys and roofs which have been observed during great earthquakes.

§ 8. The column or the wall made of pieces

When a column or a wall is built by laying piece by piece, and pieces one over another, a horizontal force acts separately upon each brick or stone, trying to displace it. If the bonding between pieces is equal everywhere, i.e. if the friction between the pieces is equal, there will be no reason for relative displacements. However, if the friction is weaker at some place, parts of the column or a wall will be moved and a crack will appear.

If the column or a wall consists of materials with different specific weights, the second moments of area of different parts will be different. The difference of the second moments of area will produce the inner forces which will cause the neighbouring pieces to be alternately compressed and moved away from each other.

This explains how is possible that during an earthquake a single brick or stone may be ejected from the wall.

Because of the different specific weights, and also because the weak bonding between the wall and the stone coatings glued to some walls, the stones are falling away during strong earthquakes. If the outer layer of stone is thin, then its falling will not cause great damage, except of the danger for people in the street. But if the layer of the stone is thick, for example if the outer half of the wall is made of stone, and the inner half of brick or some other material, then the whole wall will be destroyed. M. Baratta[6], in his book on the Messina earthquake, describes the numerous cases when the walls of the rich palaces have been destroyed, since they were coated with a thick layer of stone. In Reggio di Calabria the street corners were provided with the names of streets on the thick stone plaques. Although the plaques were fastened to the wall by means of strong iron cramps, the force of the earthquake pulled them all out of the walls.

The danger of sliding is greatest for the highest layers of stones or bricks, since the motion there is the strongest, and because, due to the weak pressure, the friction is small. If a part of the highest layer is removed, the motion is

[6] M. Baratta: La catastrofe calabro-messinese, Roma 1910.

transmitted to the next lower layers, and in this way the parts of the wall get destroyed. Therefore the greatest damage happens on the top storeys of high buildings, whereas the damage is rare in rooms at the ground level.

§ 9. The force overthrowing the column

Let us suppose that the column is placed on the base AB (Fig. 9) with no additional bonding, and that in the centre of gravity the earthquake acts with the force $M\alpha$. Let us denote the height of the centre of gravity above the base by h, and the distance between points A and C as a. The torque of the force $M\alpha$ about the axis passing through A, tending to overthrow the column, is then equal to $hM\alpha$. The torque caused by the weight $M(g \pm \alpha_1)$ of the column is $aM(g \pm \alpha_1)$ taken about the same axis. The point C will rise above the base, as soon as $hM\alpha > aM(g + \alpha_1)$.

The highest danger that the column gets overthrown will be when α_1 is negative. The condition that the column starts rotating around the axis A will be

$$h\alpha > a(g - \alpha_1)$$

or

$$\frac{h}{a} > \frac{g - \alpha_1}{\alpha}.$$

The danger that the column gets overthrown is independent on the weight of the column and depends only on the strength of the earthquake and on the height of the centre of gravity and on the area of the column base.

A column placed freely on its base will be safe against overturning only if the ratio between the height of the centre of gravity above the base and the smallest distance between the edge of the column and the normal from the centre of gravity to the base, is smaller than the ratio of the difference between the acceleration of gravity and the maximal vertical earthquake acceleration, and the acceleration of the horizontal component of the earthquake.

If one takes as the extreme case that $\alpha = 0.4g$ and $\alpha_1 = 0.2g$, there must be

$$\frac{h}{a} < \frac{0.8}{0.4} \text{ or } \frac{h}{a} < 2,$$

i.e. the height of the centre of gravity of the column must not be greater than the double distance between the edge of the column and the normal drawn from the centre of gravity to the base.

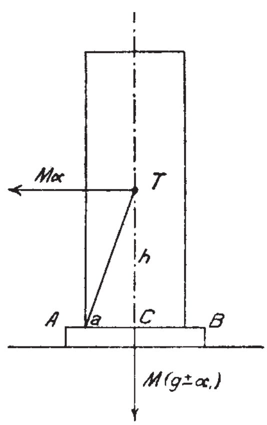

Figure 9.

In reality, the column placed on the horizontal base is in danger to be overturned even at a smaller value of the given ratio, since locally the vertical component of the acceleration can exceed the value of 0.2 g.

The thin high columns are always in danger of being overturned by an earthquake.

To be stable, a column must be strongly fastened to its base, and at the same time the base has to be solidly embedded in the ground. Assuming that the base is strongly fastened, it is easy to calculate the strength of the bond between the column and the base.

As an extreme case we take that the vertical acceleration of the earthquake is equal to the acceleration of gravity, i.e. that at that moment the column is not compressing the base. Furthermore we assume that the column is fastened to the base by means of an iron fastener which is on one side imbedded into the centre of the column and on the other side into the base. This fastener will have to withstand the whole horizontal component of the earthquake acceleration. If the weight of the column is U, the height of the centre of gravity above the base is h, the distance between the edge of the column and the normal drawn from the centre of gravity is equal to a, while the force by which the fastener is holding the column is Q, and if one considers a security factor n:

$$\frac{1}{n} Q a = \frac{U}{g} \alpha h$$

and hence $Q = n \frac{U}{g} \alpha \frac{h}{a}$.

Furthermore, if we take the extreme case $\frac{\alpha}{g} = 0.4$ and n=10, we have

$$Q = 4U \frac{h}{a}.$$

If one had U = 1000 kg, h = 100 cm and a = 25 cm, the force would be Q = 16000 kg, and the iron fastener should have the cross section of 2 cm^2. It is quite clear that, because of rusting, the iron should be made thicker than that.

§ 10. Action of the vertical component of the earthquake on the support points of the horizontal loadbearers

Just as the horizontal component of the earthquake acts on the foundation of the vertical columns, the vertical component acts on the support points of the horizontally placed beams, which can be supported only at one or at the both ends.

To simplify things, we suppose that the beam is supported on both ends, its weight is U and its centre of gravity is in the middle.

The constant weight U is divided between the two load-bearing walls and the force pressing on each of the walls is $\frac{U}{2}$. The beam is acting on the wall with a torque which depends on whether the beam is only leaning on the wall or is firmly fastened to it.

If the beam is supported only on one end and is free at the other, than it has to be firmly fastened at the point of support.

If the beam is only leaning on the walls at both its ends, then only the pressure acts, which is, as is well known, tilted with respect to the vertical direction, and tends to move the walls apart from each other.

Since the vertical component of the earthquake includes periodical increasing and decreasing of the pressure between the maximal values $U\left(1 \pm \frac{\alpha_1}{g}\right)$, the pressure of the beam on both ends will periodically rise above the normal pressure and fall below it. Thus also the distance between the walls will periodically rise and fall.

The vertical component of the earthquake is acting on the beam supported by two walls in such a way that it excites the walls into horizontal periodical vibration.

This swaying of the walls will be greater when the earthquake is stronger, and the walls can be driven apart so far that the beam is pulled out from its support sockets on the walls and falls down. This depends on the strength of the beam, for example of the girders used for ceilings, but also on the strength and height of the walls. A stronger beam will be swaying more weakly and produce a smaller force acting on the walls. When the walls are free standing and

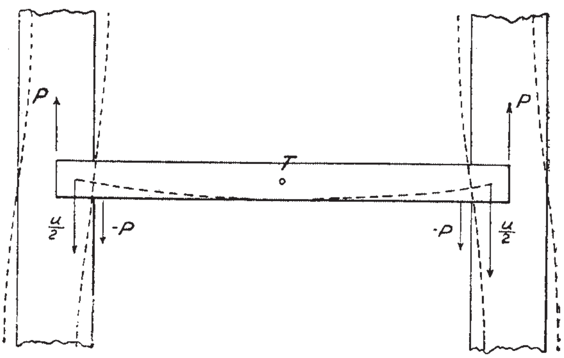

Figure 10.

less resistant, their swaying amplitude will be large and the danger of damage will increase.

If the beam is strongly fastened to both walls, they will counteract the weight of the beam and its variations by means of force couples (P_1–P), which bend the wall. Above and below the beam the walls will bend towards the interior of the building and outwards, respectively. The wall will take on the shape shown by dashed lines in Fig. 10 (exaggerated in the drawing) and the vertical component of the earthquake will act so that the bulge of the wall will periodically decrease and increase. In this case too, the result will be a horizontal oscillation of the walls.

With beams simply leaning on the wall there is the danger that the beam gets pulled from the wall and the wall breaks, either at the place where it rests on the foundation, or where it is fastened from below. The wall resists this fracture by means of its second moment of area. When beams are strongly fastened to the walls the wall resists with a double moment of area: one from below and the other from above the beam. Pulling the beam out is in this way prevented, and the action on the wall is divided into two parts, which diminishes the danger of fracture.

It follows that the horizontal load-bearers such as beams, girders, ceilings etc. should be rigidly fastened to walls, so that they form a single rigid body.

It is specially important that the bond between the wall and the beam is strong if the beam has to support large loads. This is the case with the main girder which has to support heavy walls. In calculation for these girders one also has to take into account the considerable increase and decrease of compression, both on the girder and on the walls, and it is not enough to take only the weight into account. It should be taken into account that the weight of the girder and of the wall above it can significantly increase, even up to double their values at rest.

Elastic columns

§ 11. Action of the horizontal component of the earthquake upon the elastic walls strongly bonded to the foundation

When examining the action of earthquakes on the homogenous vertical columns, one has to distinguish solid columns from the hollow ones. In the latter case we need to distinguish those columns in which different points on the same cross-section move relatively to each other from those where there is no such relative motion.

In the first class are the columns which are narrow in comparison with the length of their thick walls, in the second are those whose thickness of walls is negligible in comparison with their height.

§ 12. Vertical solid column

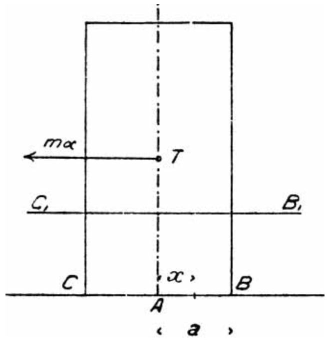

Figure 11.

The horizontal force of the earthquake acts on each particle of the column proportionally to the mass of the particle. The resultant of all those elementary forces is a certain force $m\alpha$, where m is the mass of the column. The point of application of the force is the centre of gravity of the column T (Fig. 11). The force $m\alpha$ acts in the cross-section of the column with tendency to rise the part of the column lying between the point A where the normal intersects the base of the column and the point B lying at the base on the side from which the force acts. The opposite side of the column acts by compressing the base. The force in this way tends to rotate the column around the axis passing through the point A perpendicularly to the direction of the force. The axis drawn through A has thus no tendency to rise or to plunge. If the column is vertical and homogeneous, and all its cross-sections are congruent among themselves, conclusions concerning any cross-section will be the same as those for the cross-section at the base.

Therefore we conclude that none of the points on the vertical line TA normal to the orientation of the force has the tendency to move. The part of the column on the same side from where the force acts has the tendency to stretch, but the opposite side of the column has the tendency to contract.

The plane perpendicular to the direction of forces containing the vertical line passing through the centre of gravity is the neutral plane of the column.

Let us denote the height of the centre of gravity above the basic cross--section as h, the torque produced by the force $m\alpha$ around the point A as M=$mh\alpha$, and the second moment of area of the basic cross-section BC around the axis A as

$$T=\int\int x^{2}\mathrm{d}x\mathrm{d}y,$$

where the integration is extended over the whole surface BC. In the point which is displaced by x from the neutral axis the tensile force is obtained by the known formula

$$p_x=\frac{M}{T}x$$

For the largest tension, present on the edge of the cross-section at the point B, at distance a from the axis, one obtains

$$p_a = \frac{M}{T} a.$$

If the column is cut by a plane B_1C_1 anywhere above the base, only the part of the column this plane above it will act on it. The mass of this part of the column is smaller than the mass of the whole column, and the height of the centre of gravity of this part of the column above the plane B_1C_1 is smaller than the height of the whole column above the plane BC. The torque tending to rotate the body is then smaller than the torque for the whole column. If the cross-sections B_1C_1 and BC are congruent then the second moment of area of the surface B_1C_1 is equal to that for the surface BC. The force p_x' in any point of the cross-section B_1C_1 is thus smaller than the force in the corresponding point on the cross-section BC.

For a vertical homogenous column, whose all cross-sections are parallel to the base and congruent among themselves, the greatest danger of fracture is in the plane of the base.

In the previous exposition we did not take into account the force caused by compression of the weight of the column on the base. If the weight of the column above the base BC is U and q is the area of the cross-section, then the compression on the unit of area equals $\frac{U}{q}$. As we know from previous discussion, this compression is changing and can vary between the limits $\frac{U}{q}\left(1+\frac{\alpha_1}{g}\right)$ and $\frac{U}{q}\left(1-\frac{\alpha_1}{g}\right)$.

If on the side of the force one subtracts the tension produced by the horizontal component of the earthquake, and on the opposite side adds the compression, one obtains the resultant of all forces acting at the base points:

$$P_x = \frac{U}{q}\left(1 \pm \frac{\alpha_1}{g}\right) \mp \frac{M}{T} x \frac{\alpha}{g} .$$

The positive direction of the force P_x is taken to be downwards. The sign – before the second term stands for the tension and + for the compression.

Quadratic vertical column and the pyramid

In further investigation h will denote the total height of the column above the cross-section in question. Let a be one half of the side of the quadratic cross-section. If s is the weight of a unit of volume of the column, then the weight of the column is $U=4a^2hs$, and weight-induced compressive stress upon the unit area of the base is hs.

The torque at the maximal acceleration during an earthquake at each jolt is

$$U\frac{h}{2}\frac{\alpha}{g}=2a^2h^2s\frac{\alpha}{g}.$$

The second moment of area at the cross section is $T=\frac{4}{3}a^4$. The maximal tension or compression in the point at distance x from the axis is

$$p_x=\frac{3}{2}\frac{h^2sx}{a^2}\frac{\alpha}{g}.$$

The total tension on one side and the compression on the other side is

$$p_x=hs\left(1\pm\frac{\alpha_1}{g}\right)\mp\frac{3}{2}\frac{h^2s}{a^2}\frac{\alpha}{g}.$$

The maximal tension or compression at the point a is

$$P_a=hs\left(1\pm\frac{\alpha_1}{g}\right)\mp\frac{3}{2}\frac{h^2s}{a}\frac{\alpha}{g}$$

The compression on the base increases proportionally to the distance between the cross-section and the top of the column, whereas the tension grows proportionally to the square of this distance.

For small distances, close to the top, the compression is greater than the tension caused by the earthquake, but at a certain distance from the top

$$h=\frac{2a}{3}\frac{1-\frac{\alpha_1}{g}}{\frac{\alpha}{g}}=\frac{2a}{3}\frac{g-\alpha_1}{\alpha}.$$

the tension will be equal to the compression. For larger distances from the top the tension increases, and the stability of the column is completely lost.

For a vertical quadratic pyramid, provided that the sides are not extremely slanted, the same procedure gives

$$(P_A)=\frac{hs}{3}\left(1\pm\frac{\alpha_1}{g}\right)\mp\frac{h^2s}{3A}\frac{\alpha}{g}.$$

On the tension side the compression will be equal to the tension at the distance from the top equal to

$$h=A\frac{g-\alpha_1}{\alpha}.$$

If one compares the column with the pyramid of the same height and the same volume, the base of the pyramid is equal to $A = a\sqrt{3}$. Hence

$$(P_A) = \frac{hs}{3}\left(1 - \frac{\alpha_1}{g}\right) - \frac{h^2 s}{3a\sqrt{3}}\frac{\alpha}{g}\,.$$

Comparing the expressions for (P_a) and (P_A) one is easily convinced that, except for very small heights, the pyramid is more stable than the column of the same height and volume.

When the height is small, the previous equation cannot be used anyway because of very tilted sides, so that in any case one can state that as far as the earthquakes are concerned, the pyramid is always more stable than a corresponding column of the same height and volume.

Vertical rectangular column (wall)

If the half of the width of the base is a and the half of the length of the base b, the height is h and the weight of the unit volume equal to s, the maximal tension in the direction of the width of the wall is

$$(P_a) = hs\left(1 - \frac{\alpha_1}{g}\right) - \frac{3}{2}\frac{h^2 s}{a}\frac{\alpha}{g}$$

and in the direction of the length

$$(P_b) = hs\left(1 - \frac{\alpha_1}{g}\right) - \frac{3}{2}\frac{h^2 s}{b}\frac{\alpha}{g}\,.$$

The first term in both expressions, representing the pressure of the column on the unit area of the base, is equal in both cases. The second term, which assumes that the tension produced by an earthquake is maximal, is in the second case smaller than the first one by the factor determined by the ratio of b against a.

The maximal tension the earthquake is causing at the edges of a rectangular column (wall) is smaller along the length than along the width, by the same factor by which the width of the wall is smaller than its length.

From the above equations also follows:

The action of the earthquake on the unit area along the length of the wall is independent of the width of the wall, and in the same way the action along the width of the wall is independent of the length of the wall.

§ 13. Two columns connected with braces

Imagine for simplicity that the braces have no weight and that the columns are detached along their length.

If the columns are of equal dimensions, then the tension will be equal in any cross-section intersecting horizontally through both columns. The force of an earthquake will tilt both columns equally and they will be swaying equally during the earthquake. At all intersections they will be always equally away from each other. The columns will neither extend nor squeeze the braces.

If the columns are of unequal heights and widths, the tension will exist in every horizontal cross-section, but will be different for different columns. It will be greater for thinner and taller columns, smaller for the thicker and lower ones. The first column will be bent more than the second one, and will alternately compress and extend the braces. If the braces are weak, they will have to break.

The walls of all buildings are made of columns. These columns are connected between each other by the parts of the walls lying below the windows. These connecting walls – usually built on arches spanning the space between the columns – are in their lower parts, up to the height of the ceiling, equally thick as the columns are; in their upper parts they are, as a rule very thin.

These arches, instead of linking the columns, push them apart. Due to different swaying of the two neighbouring columns, the compression between them is followed by extension, so that, if the motion is strong, the arches crack. Therefore during strong earthquakes one often sees the arches below the windows crack first.

The danger of cracked arches can be avoided only if the arches are completely abandoned and if the neighbouring columns are linked together as firmly as possible.

If the bond between the two columns is rigid, the whole wall will represent a single column. The openings in the wall (doors, windows, etc.) will not diminish its strength, but will decrease its weight, reducing in this way the torque caused by the earthquake. The tension on the base will be smaller than if there were no openings.

The greatest effects of the earthquake will be exerted in the horizontal plane intersecting the lower edges of the openings.

§ 14. Two columns connected in the middle

Such connections are some ceilings, and, as mentioned earlier, they are either just placed on the columns, or are fastened to them more or less strongly. When examining the action of the vertical component on ceilings and horizontal beams in § 4, we have stressed the importance of a strong bond between the columns and the horizontal beam. Now we shall prove that the action of the horizontal component of the earthquake also requires a rigid connection of the horizontal beams and ceilings with the columns.

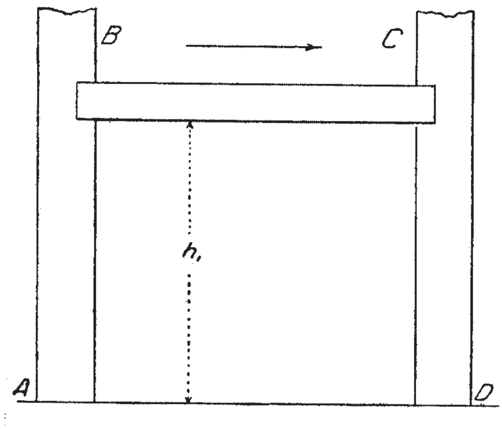

Figure 12.

a) Ceilings or beams placed on the columns

The ceiling BC (Fig. 12), whose weight per unit of length of the ceiling is U_1 and the height above the base h_1, is tightly inserted into niches on both columns, but without any other bond. Assume for simplicity that the weight of both columns per unit length is U and that the height of the centre of gravity is h. If the earthquake acts in the direction of the arrow from left to right, its action on the column AB will be almost independent of the ceiling BC. The column will be bent just as if there were no ceiling. However, on the column CD acts the ceiling with all its energy and exerts a force $U_1 \frac{\alpha}{g}$ at the height h_1. The moment of force (torque) $Uh\frac{\alpha}{g}$ is therewith increased by the torque of the ceiling, and the total torque equal to $(Uh+U_1h_1)\frac{\alpha}{g}$ acts on the column.

The column CD will be thus bent more than the column AB, whereas the edge of the ceiling near B will be pushed away from the wall. If the force of the earthquake and the freedom of motion of the columns are large, the ceiling will get pulled out of the niche near B and will collapse. When, after the next half-oscillation, the earthquake assumes the opposite direction, the ceiling will push outwards the wall AB, but at the point C it will be pulled away from the wall CD. The ceiling is thus alternately hitting one or the other wall. This explains the destruction of ceilings which has been observed during strong earthquakes. It was reported that during the large earthquake in Messina the ceilings collapsed in many homes, and from pictures of destroyed houses one can notice that the ceilings have not been fastened to the walls.

b) Ceilings or beams strongly fastened to walls

Here we must distinguish between the two cases: the ceiling is either inserted into the dents in a wall as described in a), or is rigidly connected to the walls and makes with them a single object.

In the first case, the ceiling acts with a half of its force on each column, so that both columns, if otherwise they are equally free, sway equally. There is no tendency of detachment of the ceiling from any of the two walls. Surely, if one of the walls were less free than the other, then there would be some tension and compression affecting the ceiling.

Since any wall normally has to withstand one half of the force exerted by the earthquake on the ceiling, which is per unit of length of the wall $\frac{1}{2}U_1\frac{\alpha}{g}$, it will be sufficient that the strength of the bolts is chosen in accordance with this force, adding a surplus for safety, and because of the expected deterioration of the bolts.

Normally one takes for the weight of the ceiling some 500 to 600 kg per m^2. If we take the wall-to-wall distance to be 6 m, then one meter of the ceiling weighs about 3600 kg. If we assume $\frac{\alpha}{g}=0.4$, the force which pulls the wall per one meter is 720 kg, and if we take for safety the 10 times larger value, it is 7200 kg. Per one meter, it will be enough to include one bolt having the cross-section of 0.72 cm^2. If we take into account that the opposite wall often has more freedom than the one containing the bolt and that consequently this bolt will have to bear also a part of the force by which the earthquake acts on the opposite wall, it will be safe to add to the force by which the ceiling acts also a part of the force by which the opposite wall is pulling the ceiling. It is very difficult to estimate or calculate how large this part of the force may be. In my opinion, for practical purposes it is sufficient to take one half of the force with which the earthquake acts on the opposite wall.

If, in the above example, the thickness of the opposite wall is 0.4 m and its height 4 m, its weight per meter would be 2560 kg, and the force of the earthquake would be 1.024 kg. Taking one half of this amount with a safety factor equal to 10, one would have 5120 kg. The cross-section of the bolt would have to be increased by 0.5 cm^2. Thus the total cross-section of the bolts per meter should be 1.2 cm^2.

If we take into account that ordinary iron, if it is not of exceptional quality, is hardly able to resist the force of 10 000 kg per cm^2 and that the bolts necessarily get rusted, the practical advice is to double the calculated cross-section of the bolts.

Care should be taken of the fact that, when connecting the ceilings with the walls, the tendency to rotation around the vertical axis is suppressed. Normally one places the bolts equidistantly along the length of the wall, without caring whether they are on a column or under the window. This would be correct if the wall were a homogeneous mass and the connection between the columns rigid. But, with the now widely accepted practice of constructing the walls from bricks with a poor-quality mortar, the bolts below the windows do not hold at all. The bolts placed in the columns are found in the middle of the column as well as at its edges. After each jolt caused by the earthquake, the column fastened near the edge or generally in an asymmetric manner, will bend around the vertical axis and will provoke the damage to the neighbouring connection with another column.

c) Ceilings rigidly connected to the walls

If a ceiling or a beam is strongly fastened to the columns (Fig. 13), forming with them a rigid structure, the whole system will behave as one single column or as two columns whose centre of gravity, because of the ceiling, is moved to a higher position.

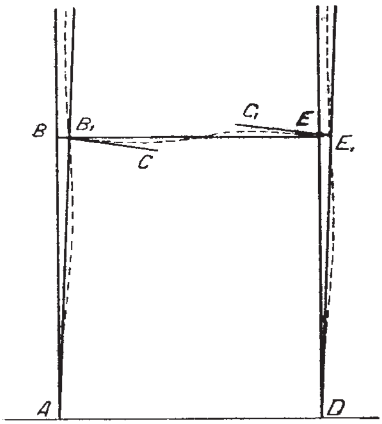

Figure 13.

Subjected to the force of the earthquake oriented, for example, from left to right, the whole system will tilt to the right-hand side. Both walls will be displaced from positions AB and DE into positions AB_1 and DE_1. Due to this displacement the couple of forces rotating the ceiling downwards should disappear at point B where the ceiling is connected with the wall. The left-hand side of the ceiling would be moved into the position B_1C. The right-hand wall, however, has the tendency to rotate the ceiling upwards into the position E_1C_1. Since it is elastic, the ceiling will act on the wall and will bend it in the opposite direction. Eventually the wall will assume the final shape as depicted, rather exaggeratedly, with the dashed lines in Fig 13.

The ceiling will assume a wavy form and the wall will protrude to the left-hand side above the ceiling and to the right-hand side below the ceiling. The action of the earthquake is in this way divided between the wall and the ceiling, reducing the danger of cracking at any point of the building.

This explains why the rigid connections of ceilings with the walls are so extremely useful.

§ 15. The walls with supporting lateral walls

In churches, large halls, houses or at the fences, thin, long and sometimes even very high walls can often be found, that are supported only at the

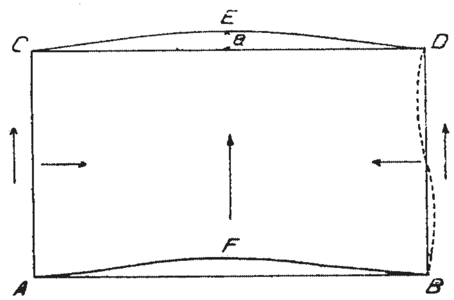

Figure 14.

ends by other walls oriented perpendicularly or obliquely with respect to the first one.

Fig. 14 shows a horizontal cross-section of such a wall at some distance above the ground. A sudden jolt exerted by the earthquake acts in the sense of the arrow. On each element of the length of the wall acts a force of the earthquake proportional to the mass of that element. Considering a unit of the height of the cross-section and that the weight of a unit of length of the wall is u, a force $u\frac{\alpha}{g}$ will then act on each unit length. The ends AB and CD of the wall, fastened to the walls AC and BD, will not move at all, while the centre of the wall which is free will move by a length a. The whole wall will be distorted and will eventually assume a curved shape AFB and CED. The strongest tension, and therefore the greatest danger of cracking, will be on the corners and in the middle of the walls.

The length a of the displacement of the middle of the wall depends on the coefficient of elasticity of the wall, on the elevation of the cross-section above the base, on the thickness of the wall and on the strength of lateral walls. The wall AB acts on the lateral walls in the following ways: a) it tends to rotate the axis of the wall BD and AC into the position depicted by the dashed line; b) it pulls, by tension, both walls to itself; c) it pulls both walls in the direction of the action of the earthquake.

At its base the wall will be left undeformed, but higher above the base the cross-section is, more deformed the wall will get.

The danger of cracking or breaking is the greatest in the middle if there are no openings on its surface. If they exist, cracks will appear at the place where the wall is the weakest.

The resisting strength of the wall depends on the quality of material of which it is built, on the compression exerted by higher layers and on the strength of the longitudinal binding, achieved with the mortar and longitudinal bolts.

To lessen the deformation, one can increase the thickness of the wall either in the middle or at some places along the length of the wall. This will, however, increase the torque, but the second moment of area will be increased even more, which will cause such a decrease in the deformation that any danger of the damage should disappear. The vertical cross-section of walls strengthened in such a way could be made conic or, even better, parabolic.

§ 16. Hollow columns with short side walls

We will assume that the walls of the hollow column are so short and thick, that the force of the earthquake does not change the horizontal cross-sections. The examples of such columns are rectangular or circular chimneys, towers etc. The whole buildings can also be regarded as hollow columns if they are built in a way which prevents the deformation of the horizontal cross-sections.

Let us first take a vertical square column with equally thick walls. If one half of the outer and inner side-lengths are a and a_1, respectively, and the height of the column is h, then the weight of the column is $U = 4(a^2 - a_1^2)hs$; the compression on the unit of area of the base hs, the torque $U = 2(a^2 - a_1^2)h^2 s \frac{\alpha}{g}$ and the second moment of area around the axis passing through the middle of the section, in parallel with one of the lateral sides, is

$$J = \frac{4}{3}(a^4 - a_1^4)\,.$$

The tension caused by the earthquake at the edge of the cross-section at the distance a from the axis is

$$P_a = \frac{3}{2}\frac{h^2 as}{a^2 + a_1^2}\frac{\alpha}{g}$$

The resultant compression on the edge of the cross-section is

$$(P) = hs\left(1 \pm \frac{a_1}{g}\right) \mp \frac{3}{2}\frac{h^2 as}{a^2 + a_1^2}\frac{\alpha}{g}$$

Comparing this expression with the one for the solid column with the same dimensions, we see that the action of the earthquake on the hollow column is weaker than on the solid column, because the denominator of the second term with the values for (P_a) is larger than it would be in the corresponding term for the solid column.

Square chimneys, towers and solidly constructed buildings are able to resist the earthquake more effectively than the solid prisms of same dimensions and made of the same material.

The expression describing the action of an earthquake on the hollow column decreases when the internal cavity (a_1) increases.

The thinner are the walls of the inner cavity, the weaker is the action of an earthquake on the column.

The walls of the column should not be too thin, because a) the thin walls are subjected to strong deformation; b) because the mortar bonding the bricks or stones in the wall does not hold at the edges but only in the middle of the contact area, as was proved by Omori. This is the reason why the second moment of area is always smaller than the second moment of area calculated previously; the tension per unit of area is thus in reality larger than has been calculated.

If the width of the walls is for example 60 cm, and the mortar does not bind 2 cm from the edge, then the actual width is 56 cm, i.e. 7% less than the one which has been measured; but if the width of the walls is 10 cm, and the mortar holds only on a surface 6 cm wide, then the real thickness is 40% smaller than the calculated one.

The same conclusions which have been deduced for the quadratic column are valid also for the rectangular and circular ones.

Analogously to the rules for the quadratic hollow column we can derive the rules for the rectangular hollow column. Let thus the half of the length of the wall be a, the half of its width b, the corresponding inner length and width a_1 and b_1, and the height of the column h.

The weight of the column in this case is $U=4(ab–a_1b_1)hs$. The torque around the base is $M=2(ab-a_1b_1)h^2 s\frac{\alpha}{g}$. The second moment of area for the axis passing through the centre of the base is $T=\frac{4}{3}(a^3b-a_1^3b_1)$.

The tension caused by an earthquake at the edge of the column will be

$$P_a=\frac{3ah^2s\alpha}{2g}\frac{ab-a_1b_1}{a^3b-a_1^3b_1}.$$

If one knows the values of a, b, h, s, a, and g, one asks whether also in this case, as was for the square column, this expression has the smallest value in the case when $a_1=a$ and $b_1=b$. With this purpose in mind, one has to examine the value of the ratio $\frac{ab-a_1b_1}{a^3b-a_1^3b_1}$ and explore how it changes when a_1 and b_1 change.

If we insert into the expression different values for b_1 between 0 and a, we do not obtain the minimal values of the ratio; if we substitute different values for a_1, we obtain the minimal value for a satisfying the equation

$$a_1^3+\frac{3}{2}a_1^2ab-\frac{a^3}{2}\frac{b}{b_1}=0\,. \tag{1}$$

Along the width we obtain the tension in the same way:

$$P_b=\frac{3bh^2s\alpha}{2g}\frac{ab-a_1b_1}{ab^3-a_1b_1^3}\,.$$

The minimal value of the ratio

$$\frac{ab - a_1 b_1}{ab^3 - a_1 b_1^3}$$

is obtained for the value b_1 which can be calculated from the equation

$$b_1^3 + \frac{3}{2} b_1^2 ab - \frac{a}{2} \frac{b^3}{a_1} = 0. \qquad (2)$$

From that follows that the rectangular column too is safer against earthquakes the thinner are its walls, but only up to the limiting dimensions a_1 and b_1 determined by equations (1) and (2). If the walls are thinner than the values obtained from equations (1) and (2), then the column is less safe against the earthquake.

Following the above derivation, the builder will be able to calculate the maximal tension in any cross-section of the building for any shape and dimension of the construction. For the horizontal force of the earthquake we have to take the fraction of the weight of the building equal to the ratio between the maximal acceleration of the horizontal component of the earthquake and the gravity, $\frac{\alpha}{g}$, for which one wishes to make the building safe. It is obvious that for the sake of safety he will have to take a somewhat larger acceleration.

If, for example, the building (a chimney) weighs 1155 tons, and we wish it to be safe against the earthquake up to the maximal acceleration of 1000 mm/s^2 [7], for safety we take α = 1500 mm/s^2. Since g = 9800 mm/s^2, approximately $\frac{\alpha}{g} = 0.15$. The maximal horizontal force of the earthquake is then 1155 × 0.15 = 173.25 tons. If the height of the centre of gravity is for example 30.85 m, then the torque caused by that force is 173.25 × 30.85 = 5344.75 t m [8].

The second moment of area of the base, which has the outer radius of 3.2 m and the inner one 2.1 m is 67.05. The maximal tension at the edge of the column is

$$\frac{5344.75}{67.05} \cdot 3.2 = 255.72 \text{ t per m}^2 \text{ or } 25.6 \text{ kg per cm}^2.$$

The area of the base is 18.31 m^2, the pressure of the gravity is 1155 / 18.31 m^2 = 63 t per m^2 or 6.3 kg per cm^2.

Calculating with the vertical component of the earthquake equal to one half of the horizontal component i.e. 0.08, in the extreme case we shall have (1 – 0.08)·6.3 kg = 5.8 kg.

---

7 mm in original (MH)

8 m in original (MH)

For the column designed in that way one would have from one side the tension equal to (25.6 + 5.8) kg = 31.4 kg per cm^2.

The column could probably resist a compression of that size, but under the tension equal to 20 kg per cm^2 the column built from bricks would certainly collapse.

If we allow the maximal tension of 4 kg with very good bricks and an excellent cement mortar then one could allow, after subtracting the compression, the highest tension of 9.8 kg per cm^2. Taking the previously given dimensions of the chimney, the maximal force of the earthquake which the chimney could withstand would be

$$x \,/\, 0.15 = 9.8 \,/\, 25.6,$$

i.e. $x = 0.057$ or, rounded, $\alpha = 570$ mm/s^2.

Assuming that the earthquake of January 2nd 1906 had the maximal acceleration of some 300 mm/s^2, the chimney designed in the described way would collapse only if the earthquake were twice as strong as the 1906 earthquake.

For our region that chimney would be probably too weak, because the disastrous earthquake of the year 1880 probably had the acceleration greater than 1000 mm/s^2.

According to the previous paragraph, when designing such a chimney we should be aware that the mortar does not hold near the outer edges of the wall. Let us assume that it does not hold at all for the nearest 4 cm. Very hot gases will flow through the chimney, probably between 300 °C and 400 °C, maybe even hotter.

In a short while the heat will weaken the mortar in such an extent that up to a certain depth it will loose all adhesive power. Let us take this depth as only 10 cm. In that case the second moment of area will be

$$\frac{\pi}{4}(3.16^4 - 2.20^4) = 59.88.$$

The maximal tension at the edge of the chimney will be, in the same conditions, 28.56 kg per cm^2, giving the resulting tension of 22.7 kg per cm^2. It follows that the chimney would be safe up to the maximal acceleration of 500 mm/s^2, and if we would allow the maximal tension of only 2 kg per cm^2, then it would be safe up to the acceleration of 400 mm/s^2.

For our situation this project would not be valid, and the walls of the chimney would have to be reinforced.

§ 17.

From the expression for the maximal tension

$$P_a = hs\left(1-\frac{\alpha}{g}\right) - \frac{3}{2} \times \frac{ah^2 sv}{a^2 + a_1^2}$$

we see that, as was the case for the solid column, at some height h the first term is larger than the second one, i.e. the compression prevails over the tension. If h would attain the value

$$h = \frac{2(a^2 + a_1^2)\left(1-\frac{\alpha_1}{g}\right)}{3a\frac{\alpha}{g}},$$

the tension would equal the compression. This height increases when the two components of the earthquake $\frac{\alpha}{g}$ and $\frac{\alpha_1}{g}$ decrease and when a_1 increases, i.e. when the walls are thinner.

For larger heights the tension is larger than the compression, so if we take the height large enough, the tension will be great enough to break the column.

Let us consider for an example an ordinary house chimney for which a = 22 cm and a_1 = 7.5 cm. Let the maximal horizontal acceleration on the roof be twice as large as the one at the centre of gravity of the building. For an earthquake of medium strength with $\frac{\alpha}{g} = 0.1$ and $\frac{\alpha_1}{g} = 0.1$, h = 147 cm, and if the horizontal force were twice as large, $\frac{\alpha}{g} = 0.2$, it would be h = 74 cm.

Our ordinary solidly constructed chimneys are safe from the weaker earthquakes up to the height of 147 cm, and for stronger earthquakes they would be safe up to the height of 74 cm.

However, even lower chimneys on our houses can be prone to breaking and collapsing even for earthquakes weaker than $\frac{\alpha}{g} = 0.1$.

1. Due to the fact that our high wooden roofs are easily movable, especially lengthwise, they can be set into motion much stronger than is the motion of the building itself. The roof can thus act on the chimney with a force much greater than is the force by which the earth is acting on the building.
2. When the chimney starts moving forward, it crushes the mortar placed between itself and the roof, so that after a few oscillations the building cracks between the chimney and the roof. At first, as soon as the crack forms, the chimney starts swaying freely, but at the next oscillation it hits the roof and breaks. This happens especially where the chimneys are built on the attic floor and erected vertically through the roof.

3. It can happen that the chimney built on the attic breaks initially in the space between the attic floor and the roof, and only after that above the roof.

It follows that the chimneys should be built as thin and as strong as possible, and that some other material, apart from bricks, should be used.

§ 18. Columns having large horizontal dimensions

The rules derived for the hollow column are valid only if every deformation of the horizontal cross-section of the column is excluded. Since the hollow column shows an extremely high resistance against the earthquake, and is even more resistant the thinner are its walls,

and since all our buildings are actually the hollow columns, it is of greatest importance that they are built in such a manner that every deformation of the horizontal cross-section is prevented.

The stability of the cross-sections can be obtained:

1) if at the equal distances of height each of the four sides i.e. every wall of the column is fixed so that the deformation of its horizontal cross-section becomes impossible.

In a method of building practiced in Zagreb, an iron bar equally long as the wall, is placed in the middle of the wall at each storey level. It is fastened at its ends either to the wall or to the next bar, placed in the middle of the next side (wall). The purpose of that bar is to strengthen the wall lengthwise.

The only benefit of this bar is to prevent the tension which could emerge, due to the unequal bending of the unevenly high and wide columns of which the inhomogeneous wall is made. This should prevent the cracking of the lintels above the windows.

If these bars fulfil that task or not, depends on their strength and on the way how its parts are fastened to each other. If this connection is weak, and it usually is, then these bars will be useful only during very weak and moderately strong earthquakes, but would hardly help in strong earthquakes. If the homogeneous wall is constructed, and the arches below the windows are replaced by girders, then the girders and the strength of the material itself will bind the columns together as if they were all a single column.

In that ideal case the horizontal fastenings along the wall would be totally unnecessary.

Since no wall is built so ideally that it could be taken as a single homogeneous unit, the horizontal braces along the length of the wall are useful. Since every wall is built of columns, and different columns have usually not only different lengths but often also different heights, each wall will oscillate separately both in the sense of height and in the sense of width (thickness) of the wall. The most unfavourable case will happen when one half of the columns starts moving in one direction and the other half in the other. The wall will easily withstand the squeezing starting at some moment, but it will not be able to resist the stretching. This will be the task of the brace. *The brace thus has to be strong enough to withstand the highest acceleration arising when two halves of the wall move in opposite directions.*

Let T be the weight of a whole storey (because the brace has to hold one half of the storey beneath itself and one half of that above), and that the maximal acceleration of the horizontal component of the earthquake is α, so that the maximal force of the earthquake is $T\frac{\alpha}{g}$ kg. The brace must have such a cross-section that it would be able to withstand the force $T\frac{\alpha}{g}$.

For our purpose let us take $a = 0.2\,g$, so that the brace should be strong enough to bear the load equal to 0.2 times the weight of the wall. If the wall is 4 m high and 60 cm thick, then its weight per one meter is $40 \times 6 \times 10 \times 1.6$ kg = 3840 kg. The cross-section of the bar should thus be 0.76 cm^2 per one meter of the wall. This is valid for a safety factor of 10. Since for this case, which is anyway rather infrequent, even five times larger safety will be sufficient, than the cross-section of the bar need not be smaller than approximately 0.4 cm^2 per one meter of the wall. For a wall 15 m long on the ground floor, where the wall is the thickest, one would need the bar with the 6 cm^2 cross-section, and for a wall 30 m long the bar should have the cross-section of 12 cm^2. On higher floors the weight of the walls is smaller but the freedom of motion is greater, so that the same thickness would be necessary as on the ground floor.

The bars, as they are nowadays placed in the walls in our buildings, hold nothing at all, and are only a redundant expense because: 1. the pieces of which the whole bar is made are only hooked one to the other, and the joint possesses a great freedom of motion so that the wall will break before the bar becomes effective; 2. the bar is not highly strung but is sinuous, so it has first to straighten to be able to act; in the meantime, the wall has already been broken; 3. the bar is not strongly fastened to both ends of the wall, and even if it starts being effective, it will only provoke damage to the corners of the building.

The bars placed into the wall only slightly reduce the horizontal deformation of the wall, because they are too weak and have inadequate shape. To prevent the horizontal deformation, the wall should be strapped from outside with the girders having the H-shaped cross-section. They would use all their strength to suppress any outward deformation and would compress the wall towards the inner beams, so that the deformation in both senses would be hindered.

A much more convenient method of preventing the horizontal deformation is to make the ceilings:

1. rigidly connected with the walls;
2. built so strong that, according to their main purpose, they do not allow vertical deformation;
3. resistant against the horizontal deformation in any direction.

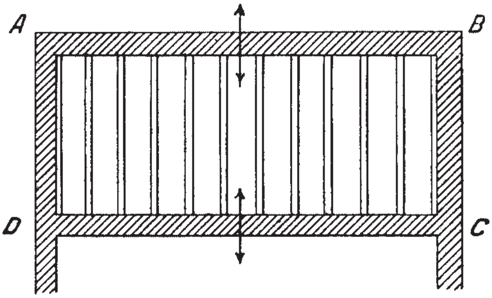

Figure 15.

If in this way the horizontal deformation of the walls of the building is avoided, then the whole building can be treated as a single homogeneous column. In that case any bending in the direction of the smallest dimension of the wall becomes converted into tension, and against the tension act the gravity compression as well as the strength of the mortar and other building materials.

In our parts the ceilings are usually made so that the two nearest walls (Fig. 15) AB and CD get connected with beams, placed equidistantly, with no regard whether they are placed on the columns or below the windows. Usually every other or third beam is linked with the wall by means of an iron brace. The planks together with some sound-absorbing material are then placed on the beams, and above it all comes the floor.

Such a construction connects two opposing walls, but allows an almost complete freedom of motion of the middle of the wall perpendicularly to the orientation of the walls AB and CD, because both walls are swaying together. The walls AB and CD are usually completely free and can move in any way. Very rarely these two walls get connected with the neighbouring beams in the middle.

In the earthquakes during which all walls get deformed, as is usually the case, the cross-section of the building ABCD gets deformed (Fig. 16). The mid-

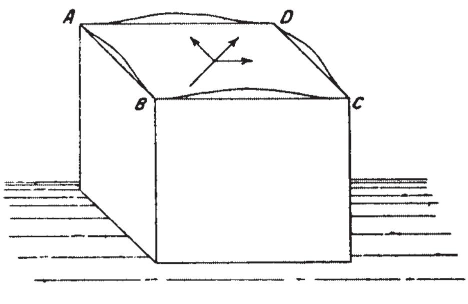

Figure 16.

dle parts of the walls are bent, and they do not add to the resistive strength of the building; just the contrary: the neighbouring walls have to support them to prevent their toppling. If this deformation is hindered, then the shape of the cross-section hardly changes at all, and the building becomes a monolith.

When investigating such a solid structure, it will be possible to calculate the tensile and the resistive strengths in the following way:

Imagine the building which has the rectangular base, with the half of the length a, the half of the width b and the height h. At some heights h_1, h_2, h_3, etc. lie the ceilings and the roof, having the weights Q_1, Q_2, Q_3, etc. Let us define the thickness of the walls by the half of the inner length and width – a_1 and b_1, respectively.

The compression on the unit of area of the base is

$$\left[sh+\frac{1}{4(ab-a_1b_1)}(Q_1+Q_2+\ .....)\right]\left(1-\frac{\alpha}{g}\right).$$

The torque, caused by the earthquake with acceleration α, will be equal to the sum of torques of the walls and the ceilings

$$M=2(ab-a_1b_1)h^2s\frac{\alpha}{g}+(Q_1h_1+Q_2h_2+\ .....)\frac{\alpha}{g}.$$

The second moment of area in the base plane along the direction of the length a is

$$T=\frac{4}{2}(a^3b-a_1^3b_1).$$

On the edge of the base the earthquake will exert the tension

$$P=\frac{3a\alpha}{2g}\frac{(ab-a_1b_1)h^2s+(Q_1h_1+Q_2h_2+\ ...)}{a^3b-a_1^3b_1}$$

If we subtract the tension from the compression of the weight of the building, we obtain the remaining compression per unit of area on the edge of the base.

$$\Pi=sh\left(1-\frac{\alpha}{g}\right)+\frac{1-\frac{\alpha_1}{g}}{4(ab-a_1b_1)}(Q_1+Q_2+\ldots)-\frac{3a\alpha}{2g}\frac{(ab-a_1b_1)h^2s+(Q_1h_1+Q_2h_2+\ldots)}{a^3b-a_1^3b_1}$$

or

$$\Pi=sh\left(1-\frac{\alpha_1}{g}\right)-\frac{3a\alpha}{2g}\frac{(ab-a_1b_1)h^2s}{a^3b-a_1^3b_1}+\frac{(Q_1+Q_2+\ldots)}{4(ab-a_1b_1)}\left(1-\frac{\alpha_1}{g}\right)-\frac{3a\alpha}{2g}\frac{Q_1h_1+Q_2h_2+\ldots}{a^3b-a_1^3b_1}$$

The first two terms represent the action of the walls, and the other two the action of the ceilings.

As mentioned above, the action decreases with the reduction of the thickness of the walls. If we include the reduction of the torque due to the presence of openings in the walls (windows, doors, etc.) and if we calculate the tension at the lower end of the windows and the corresponding decrease of the second moment of area, as the result we obtain a certain small thickness of the wall, satisfying the condition that the tension should not exceed a certain limit.

The action of the ceilings and other loads a building has to withstand consists of the compression on the base plane oriented in the direction of the force of gravity, and of the tension. This part of the equation also requires some minimal walls' thickness determined by the maximal resulting tension.

If we take for example a kind of building usually met in practice, we have to perform the calculation in the following way.

Taking given external dimensions and the general building plan into account, we first define some minimal thickness of the walls. Based on these data we derive the pressure on the unit of area, the torque and the second moment of area for the lower end of the window on the highest floor. By calculating the second moment of area we have to take into account that at both outer and inner wall the mortar doesn't hold up to the depth of 3 to 4 cm, and that the outer and the inner dimension will be reduced and enlarged, respectively, by this amount.

The vertical component of the the earthquake force is constant for the whole building, and the horizontal one is greater in the higher storeys than at the centre of gravity of the building. It is not known by which amount one should increase the horizontal component of the force of the earthquake for particular floors, because it is not possible to obtain it by calculation, neither was it investigated experimentally. The builders will do well if they assume twice the value of the force at the centre of gravity.

The earthquake of year 1880 quite certainly had the maximal acceleration larger than 1000 and smaller than 2000 mm/s^2.[9] Since in our region the disastrous earthquakes with accelerations much larger than 2000 mm/s^2 are unknown, and it is not financially reasonable to build a building able to withstand the earthquake which will probably never happen, it will be sufficient to take as basis of the calculation 2000 mm/s^2 for the centre of gravity, or $\frac{\alpha}{g} = 0.2$. The vertical component of the earthquake is always smaller than the horizon-

[9] For an event like the 1880 one (M_L = 6.2, epicentral distance D = 15 km, depth of the hypocentre h = 10 km), attenuation relationships used today in Croatia yield expected peak horizontal accelerations between 0.15 and 0.20 g! (MH)

tal one, and it will be enough if we take in the calculation $\alpha_1 = 1000$ mm/s^2 or, as a rounded value, $1-\frac{\alpha_1}{g}=0.9$.

For all storeys we shall take $1-\frac{\alpha_1}{g}=0.9$, and 0.4 for $\frac{\alpha}{g}$ on the highest storey, if the building is very high; for lower storeys we shall take 0.3 and 0.2.

With these data the builder will be able to calculate the maximal resultant of the tension and will learn whether the assumed thicknesses of the walls are too large or too small. In the first case the thickness will have to be reduced, and in the other increased, and, if necessary, the calculation should be repeated.

The same calculation should be performed for all the weakest horizontal cross-sections of the building.

When the building is made of timber or reinforced concrete, the thickness of the walls can be made arbitrary. For bricks, their dimensions determine the lowest thickness and all the other greater ones. Our bricks of 30 cm allow the increase in steps of 15 cm, whereas the smaller bricks of 22 cm, for example, allow smaller intermediate thickness.

It would be very convenient if the simple models of buildings representing the most common ones nowadays constructed in our region, would be used for experimenting with the resisting strength. In spite of the cost of such these experiments, the results would be extremely useful, and would yield the smallest thickness of the walls necessary for our buildings. At the same time, these experiments would help to find whether it is necessary to replace the usual ordinary mortar with the mortar made with the cement, and would it be financially acceptable or not. Some examples, for which I performed the calculations myself, show that the thinner walls executed with the cement are stronger and cheaper than the thick ones. I will quote one among these examples:

The construction of the tower of a village church is planned, 20 m high from the ground to the pyramid; the pyramid has to be made of wood and strongly fastened to the walls beneath it. The length and width of the quadratic cross-section of the base are both 4 m. There are three propositions: 1. The ordinary wall 45 cm thick from the base to the top; 2. a similar wall but with the cement mortar; 3. the wall 30 cm thick with the cement mortar. Let us assume that, as usually, there are so few windows on the tower that they can be neglected in the calculation.

Let the pyramid, the ceilings, bells etc. have the total weight of 46 tons, with the centre of gravity at the height of 22 m.

The calculation for the base at the ground-level will be as follows:

1. For the wall with the ordinary mortar:

The area of the base with the door subtracted is 6.6 m^2. The volume of the walls is approximately 140 m^3, and the weight 224 t. Adding to it 46 t mentioned above, one obtains the compression on the base equal to 4.1 kg per cm^2, which is allowed by the construction codes.

For the given data and the ratio $\frac{\alpha}{g} = 1$ the torque of the wall and of other loads is equal to

$$M = 224 \times 10 + 46 \times 22 = 3252 \text{ tm}$$

The second moment of area for the cross-section at the base level for the neutral axis parallel with the door is (the rounded value): $T = 10.6$.

The maximal tension on the outer edge of the base is approximately 61.4 kg per cm^2.

If, for the moment, we neglect the vertical component of the earthquake, by subtracting the compression from the tension we obtain the resulting tension of approximately 61.4 kg – 4.1 kg = 57.3 kg.

As the brick in the ordinary mortar does not allow any tension, the tension of the earthquake can be at most 4.1 kg. The torque of the force of the earthquake should thus be reduced, in order to get from these 61.4 kg only 4.1 kg, i.e. 61.4 × 0.067 = 4.1, and from that follows the force of the earthquake $\frac{\alpha}{g} = 0.067$.

If we account also for the vertical component of the force of the earthquake with $\frac{\alpha_1}{g} = 0.06$, the horizontal component is reduced to the value $\frac{\alpha_1}{g} = 0.065$. Assuming a good quality of the material and construction, the tower will be safe up to the maximal acceleration of the earthquake of approximately 650 mm/s^2, i.e. up to the force twice as large as that of the earthquake of January 2$^{\text{nd}}$ 1906.

2. For the tower with the cement mortar we can confidently use the maximal resulting tension of 4 kg per cm^2. Adding to this tension the compression of 4 kg, we obtain 8 kg for the tension of the earthquake, and 8 = 61.4 × 0.13.

The tower built in this way could resist an earthquake nearly equal to the one of the year 1880.

3. If the walls are only 30 cm thick, then the volume of the walls is 88 m^3, and the weight is about 140 tons. The area of the base, with the door subtracted, is 4.14 m^2. The compression on the base is (140+46)/4.14=4.5 kg per cm^2, thus, due to the opening for the door, slightly larger than in the first case.

The torque will be

$$M = 140 \times 10 + 46 \times 22 = 2412 \text{ mt.}$$

The second moment of area will be, rounded, T=11.

The maximal tension at the edge is

$\frac{2M}{T} = 4824:11 = 439$ tons per m^2 or 43.9 kg per cm^2 of the area of the base.

The largest tension allowed, taken together with the compression, is 8.5 kg.

The torque or the maximal tension should be multiplied by 0.195 to get the tension of 8.5 kg, so that the tower will be able to resist the maximal acceleration of approximately 2000 mm/s^2, i.e. one and a half times larger than if its walls were 45 cm thick.

If we account for the price of the walls with cement mortar, which is 30 % higher than for the ordinary wall, the wall of 45 cm with the cement mortar is 30 % more expensive than the wall of the same thickness with the ordinary mortar. The wall 30 cm thick with the cement costs 13 % less than the wall of 45 cm with the ordinary mortar. Taking the cement mortar and reducing the thickness of the walls, we are economizing and at the same time we obtain a tower 13 % cheaper and three times stronger against an earthquake. This simple example can be easily generalized and we could calculate how much more would a building with the walls of reduced width cost, if one were using the cement mortar and the rigid ceilings. *With insignificantly higher costs, at most up to 10 %, we would have a building three times stronger, able to withstand without damage our strongest earthquakes.*

The reduction of the wall thickness, however, is not feasible, as long as the construction codes remain unchanged: they define the minimal thickness of the wall made of bricks, without regard to the type of construction.

Before it would be possible to change the construction code, one would have to prove experimentally what I just proved here theoretically. It would be thus of major importance to make experiments with the ordinary buildings and examine the limits to which the thickness of the wall can be reduced. I have listed here all possible reasons for reduction of the wall thickness, but also those which put limits to that reduction. This is a question which can be resolved only by experiments.

In the introduction to the present work I have already explained of what huge economic value would be if one could, in our country, construct buildings for which one would know in advance that they are safe against earthquakes. Now I have to add that, according to the principles exposed here, this is feasible, only the theoretical conclusions should be experimentally proved. As the principles are irrefutable, so are the conclusions, but they include the assumption that the materials, are immaculate within certain limits. Thus, the purpose of the experiments is not to test the theoretical conclusions, but to answer whether these conclusions could be applied to our building materials and to our methods of construction.

There is another argument against the reduction of the thickness of the walls. If the thickness of the wall is considerably reduced, both heat and frost will much more easily penetrate into the interior of the building. We shall have large variations of the temperature within buildings, and it will be difficult to keep rooms warm in the winter. This can be easily repaired without increasing the thickness of the walls: it is enough to make the walls hollow. Such a construction, provided that the cement mortar is used, is easily executed, and the building with hollow walls would be only slightly more expensive, or even not at all, than the construction of the normal wall with the equal horizontal cross-section. The space between the walls is filled with air, which is poorer conductor of the heat than the brick. Against the earthquake such a hollow wall is by itself much stronger than the equally thick full wall, and also the cavities in the walls of the building reduce its torque, at the same time hardly changing the second moment of area.

Vertical columns attached at the lower and upper ends

§ 19.

The conclusions given in § 8, where the action of the vertical component of the earthquake on the horizontal beam is discussed, can be applied to the present case if one simply rotates the whole situation by 90°. On the upper base of the vertical column acts the compression coming from walls and objects above the column. On the lower base, besides those, acts also the weight of the column itself. If we wish that the compressions on both sides be equal, the lower base should be larger than the upper one, or the column should have the conical shape, wider in the lower part. If one takes into account the vertical component of the earthquake, the compression on both lower and upper base planes will be changing.

The horizontal component of the earthquake acts proportionally to the mass of the column ($M\alpha$), and is smaller for thinner walls. This force acts in the horizontal direction on both supporting points, trying to move the column from its position. If the column is just placed upon the base, and the upper load only placed on the column, then it can happen that the column moves from its place or is even overthrown. This is especially probable if there is a strong vertical component of the earthquake, reducing the compression on both bases of the column.

Therefore it is necessary that the column is strongly fastened at both upper and lower base.

If the column cannot be moved, then it will push its base and whatever is on it.

Since the force of the earthquake acts on each particle proportionally to its mass or weight, during the earthquake every column will bend in the direction of the force. If we know or can estimate the maximal acceleration of the earthquake, it is easy to calculate the tension in the middle of the column at its both ends, and using the results thus obtained, to determine the dimensions of the column which could then withstand the earthquake of some strength. The greatest danger of breaking is in the middle of the column. Therefore it is advisable that the column is made thicker in the middle than at the lower and upper ends.

Hollow column attached at lower and upper ends

§ 20.

Whether the solid column is stronger or weaker than the hollow one, can be found out if one explores how much larger or smaller is the tension at the edge of the hollow column in comparison with the solid one, provided that both columns are equally resistant against the vertical compression, i.e. that they have equal horizontal cross-sections.

Let us consider a hollow column having the external radius equal to r_0, the inner one r_1, and solid column with the radius R. Since the areas of the two cross-sections are equal, it must be

$$R^2 = r_0^2 - r_1^2.$$

If $r_0 = Rx$, with x greater than 1, then $r_1 = R\sqrt{x^2 - 1}$.

The tension at the edge of the hollow column is $\frac{2r_0h^2s}{r_0^2 + r_1^2} \cdot \frac{\alpha}{g}$, or if we substitute the given values for r_0 and r_1, it is $\frac{x}{2x^2 - 1} \cdot \frac{2h^2s}{R} \cdot \frac{\alpha}{g}$. The tension at the edge of the filled column with the radius R is $\frac{2h^2s}{R} \cdot \frac{\alpha}{g}$.

When we compare these two expressions, we see that the first one is smaller than the second one for all values of x greater than 1. The ratio $\frac{x}{2x^2 - 1}$ is the smallest for $x = 1$.

Each hollow column, having the area of the horizontal cross-section equal to the cross-section of a given solid column, is stronger than the solid column.

It follows that also the walls of the buildings, especially the columns between the windows, should be made hollow, because the strength of the building is thereby increased.

If, because of the climate, it is not advisable to reduce the presently accepted thickness of the walls, then the walls may be made hollow and the strength of the building will be increased. The cavity within the wall will be a poor conductor of heat, and the smaller total thickness of walls will allow for a better ventilation.

Supposing that the cement mortar will be used, the walls might be built as follows:

1. The thickness of the wall could be the same for the whole building, or, for some very high buildings, it could be somewhat smaller on the highest storeys.
2. To the the ground-floor, i.e. up to the height of approximately 20 cm above the ground-floor level, the wall should be solid.
3. At the level of each storey there should be a horizontal belt of the solid wall, approximately 1 m high, which would enclose the whole building by a strong horizontal bond and which would be rigidly connected to the ceiling.
4. In between two such belts, i.e. within the height of each storey, the wall should be hollow. The volume of the corresponding cavity should be the largest on the highest floor and should decrease as the storeys get closer to the ground-floor.

The cavity in each of the columns should be divided into numerous vertical cavities, so that the wall would actually consist of a row of the columns with the H-shaped cross-section. This would give to the wall the greatest possible strength.

The construction of such a wall would not require much more work than would be necessary for the solid wall with equal area of the reduced cross-section. This would reduce also the cost of the wall.

Buildings with attached wings

Fig. 17 shows the cross-section beneath the roof of such a building. The earthquake is acting in the direction of the arrow at the moment when we observe the building. The action of the earthquake is divided into two components having directions of the main walls.

The action along the length of the main building, oriented from the left-hand to the right-hand side:

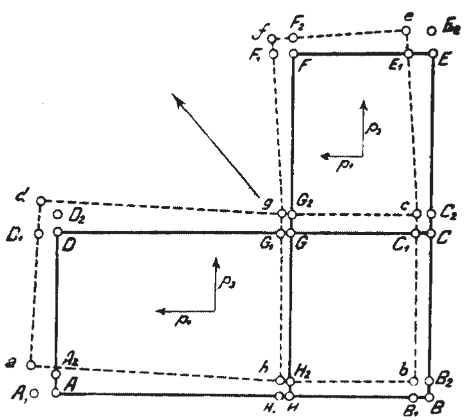

Figure 17.

The resistance of the main building along the orientation of the length (p_1) is very large, thus the movement of the corners A B C D along this direction will be minimal, i.e. to the points denoted as A_1 B_1 C_1 D_1 G_1 H_1 (distances on the figure are exaggerated). The attachment CEFG is freer in its motion than the main building, because the shock acts on it along the width. The corners E and F will move further than the other corners, for example to points E_1 and F_1.

Along the direction p_2 the main building is much more free than is the attachment. The corners A and D will be moved to A_2 and D_2. The corners B_1 C_1 E and F_1 G_1 H will be moved much less than the corners A and D, and will come to the points B_2 C_2 H_2 and F_2 G_2 H_2.

The new position of the corners, assuming no bending of the walls, will be a, b, c, d, e, f, g, h.

The shock affects mostly the corner G; other corners are affected less, corner B the least. If the shake changes orientation, the corner g will again be stretched the most.

Of all the joints between the main building and the attachment, mostly affected is the inner corner, which can easily break during an earthquake.

When the earthquake hits in the direction p_1, the wall CD get moved, somewhat stronger than the wall AB, and in the same way, the hit coming from the direction p_2 moves the wall FH somewhat more strongly than the wall BE. This provokes also the deformation of the cross-section of the wall BCGH.

During an earthquake, the free ends of both wings AD and EF are submitted to the strongest motion, whereas the strongest tension appears in the walls FH and CD.

The walls FH and CD should be stronger than other walls, and the corner C should be the strongest corner of the whole building.

Buildings in a row

If the two buildings are adjacent to each other, then the foundations of both buildings exhibit the same vibrations during the earthquake. At all higher levels, however, each building vibrates in its own way. If the two buildings are joined together, then both the tension and the compression of one building on the other will be present in all directions. If the bond between buildings is weak, or if the freedom of motion of one building is much larger than that of the other, a strong earthquake will first break the bond at the uppermost floor level, where the difference between the motions is the largest.

It is dangerous to firmly join buildings of very different dimensions and of different strengths, especially if they are high.

If the buildings are simply placed one by the other, with no mutual bond, there will be no obstruction to free motions of both buildings along the separating plane. Perpendicularly to this plane, the motion outwards will be free, but when returning, if they are too close, the houses will collide.

Because of this, the buildings which are close to each other should be constructed in such a way that there is no distance between their foundations, but starting from the foundations up to the top the distance between them should increase.

Taking into account the amplitude 3 to 4 cm of our largest earthquakes, the roofs of the two buildings should be some 10 cm apart. Equally the high bell-towers of our churches should not be closely leaning to the church facade, but should be detached from it. The walls are too weak to withstand the strong motion of a heavy tower.

Therefore the greatest damage occurs where the tower is leans to the wall, or where the wall itself is at the same time one wall of the tower.

The protuberances on the buildings

It remains to discuss the protuberances on the buildings, whose dimensions – compared with the dimensions of the building – are so small that one can not talk about separate additions or wings.

Small protuberances can go up from the foundation or can exist only within one single storey. The ending parts of such a protuberance can be leaning on the main transversal walls, or can simply rest on the vertical walls. The base plane of such protuberance can be either the foundation of the building, or any of the ceilings. The principal wall, on which the protuberance is found, can either be unaffected, so that one can enter into it only by a special door, or the main wall can be missing along the whole height and width of the protuberance. During an earthquake such protuberances are subjected to stronger motion than the wall on which they are leaning; therefore they act on the wall and on the whole building stronger as their weight increases. They cause: 1. extension of the walls to which they are attached; 2. bending of the walls; 3. rotation of the corners; 4. stretching and rotation of the ceilings. Besides, there is also a relative motion of the sides of the protuberance in respect to walls with which they are in contact.

In our parts these protuberances should be avoided, and all details which distort the symmetry of the building should be left out. The protuberances should be as lightweight and as strong as possible, and be very tightly fixed along the contact line with the main wall. Great is the danger of the protuberances whose ending walls are not supported by the strong transversal wall of the building, i.e. of those which are placed in the middle of the walls.

Such protuberances, in the form of the oriel windows, are nowadays very fashionable, and both architects and authorities should strictly control their construction, to avoid mistakes which could prove be dangerous for the buildings in an earthquake, and pose threats for human lives. In a strong earthquake such windows or their walls might be damaged and cause numerous casualties.

Internal separating walls within a building

Such walls come in two kinds: they either strengthen the building and bear the load of ceilings, or just divide the space and separate the rooms.

1. Internal (middle) walls of the building strengthen the whole building, and the more there are, the building is closer to an ideal hollow column. They increase the torque of the building and its weight. Their resistive power to the shaking of an earthquake perpendicularly to their vertical surface is very small, because they are much closer to the neutral plane than the main walls. However, the resistance to impacts along their length is quite large, and increases the resistive power of the whole building. Besides, more there are strong lateral walls in the building, stronger are the ceilings.

Since the purpose of the internal walls is to bear the load of the ceilings and to increase the stiffness lengthwise, it would not be good if these walls were too thick. If it is recommended that the outer walls are better when thinner, even more is this recommendation valid for the internal walls.

2. Separating walls. Their only task is to divide the space. They should be as lightweight and as strong as possible, and should be very strongly connected with the main walls.

The area of such walls is usually large, and as they are not supported in the middle they are often collapse in strong earthquakes. Therefore it is necessary that these walls have reinforcements fixed to the floor and the ceiling at the door openings. In this way the wall would be divided into three parts. Our separating walls are too thick, and the building material is too heavy.

§ 21. Conclusion. Fundamental rules for the construction of buildings which should be safe against the earthquakes

So far we have considered the action of a single thrust on the whole building and its separate parts, and have drawn certain general conclusions.

1. It is not possible to construct a building which would be safe against all possible effects of a catastrophic earthquake.

If the ground beneath the building cracks, the building on it must crack too. If the ground collapses, so must the building; if the steep short waves develop on the earth's surface, the building will either crack or be destroyed.

2. By using common material a building can always be constructed which will resist all, even the strongest, earthquakes which occur in our country (up to the maximal horizontal acceleration of 2000 mm/s^2).

This goal may be accomplished if the building is made as strong as the construction material allows. Since the reinforced concrete is the only material from which the building can be made as a monolith, and since with all the other material it must be made of smaller or larger pieces, one has to try to connect these pieces between themselves as strongly as possible and to obtain the building which can be at least approximately regarded as a monolith. There are following rules:

1. On the steep slopes, especially on the upper edge of a steep incline no building should be built.

2. The soil at the site where the building is planned should be carefully examined before the construction starts. If necessary, the ground should be artificially reinforced.

3. The foundation of the building should be strong and thick, in order to make the compression on the ground per unit of area as small as possible.

If the normal compression of the wall on the foundation is k kilograms per cm^2, and the building is constructed in such a way that, to be still safe at the strongest earthquake, allows the tension or the compression of 4 kg per cm^2, then the maximal compression on the foundation will be $k + 4$ kg per cm^2. The lower surface of the foundation which is placed on the earth has to be made so large that $k + 4$ kg pressure on the foundation corresponds to at most 2.5 kg per cm^2 pressure on the surface of the ground. The harder and more uniform is the ground below the building, the larger can be the allowed compression per cm^2.

4. The foundation of the building should be a monolith, best made of concrete (large pieces of rocks are allowed in concrete). If the builder has any doubts concerning the quality of the ground, he should reinforce the foundation wall by inserting into it lengthwise a long and strong iron bar.

Placing the building on a single concrete block is unnecessary, except in some special cases. If the block is especially large, then this is an expensive luxury; if it is thin, it can even be dangerous for the building. If the block is too thin, it will be carrying the building without cracking until the first strong earthquake, but then the block will crack, the walls will settle abruptly, the building breaks and a great damage can follow.

The foundation wall must not only be thick but also deep enough, to prevent its horizontal displacement during earthquakes. The depth and the foundation of the wall will be determined by the quality of the ground.

If below the whole building there is a cellar, and if the ground is good and the underground wall thick enough, it is not necessary to add another special foundation wall. The building of the girls' lyceum (formerly real gymnasium) in Zagreb is built on the homogeneous hard clay and although it has no foundation, and is very high in comparison with its width, it successfully resisted the earthquake of the year 1880. There are many such buildings in Zagreb.

It is advisable that the building is a monolith if possible. The masonry with brick and stone in the ordinary mortar, with little lime and a lot of sand does not allow any tension prevailing on the compression. The building made with the ordinary mortar cannot be a monolith, and is always weak in an earthquake. If we wish to construct a building which would be safe against earthquakes, the ordinary mortar must be replaced by the good mortar with cement. In that case we can calculate the tension and the compression by which the earthquake acts in any point of the walls, obeying the rules valid for the hollow column.

5. If the building is a monolith, and it is constructed so that it can be regarded as a single hollow column, the thinner are its walls, more resistant against the earthquake it will be. The thickness of the walls then depends only on the load it will have to bear i.e. on the weight of ceilings, roof and the other loads. By calculating the thickness of the walls the weight of the dividing walls need not be taken into account.

6. In order that the building be considered a single hollow column, all the walls have to be built so that any deformation in the horizontal direction should be hindered as much as possible.

This can be obtained by putting rigid roofs on buildings and that both the roof and the ceilings are firmly connected with the walls.

7. Since one of the most important walls is the firewall, it has to be equally solid as all the others, and equally strongly connected with the ceilings and the roof.

8. More transversal strong walls the building will have, the stronger it will be.

9. All the arches, wherever possible, should be replaced by the transversal beams, because the arches stretch the walls, and the beams bind them together.

10. All parts of the building which do not serve to increase its strength should be reduced to a minimum. The stairways should be as light as possible, and the same is valid for the separating walls. Any unnecessary heavy ornaments especially on the roof should be removed.

11. Attachments and wings should either be strongly connected to the main building (taking care about the inner corners), or should be built as separate independent buildings.

12. Various protuberances on the building, if they are really necessary, should be linked with the transversal walls and be made as light and strong as possible.

13. The roof should be light, strong and, as stressed in paragraph 6, tightly connected with all walls. The flat roofs are especially recommended for the regions often subjected to strong earthquakes.

14. The chimneys must be light and strong in the vertical direction. Where passing through the roof, they should be strongly connected with it. The

height of the chimneys above the roof must not exceed 60 cm. If the higher chimneys must be built, their top must be tied to the roof on all four sides.

15. Iron railings should be mounted on the edges of the roof to prevent the roof tiles from falling into the street during an earthquake.